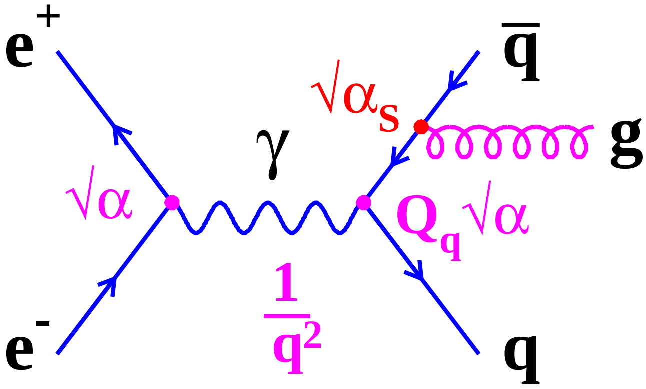
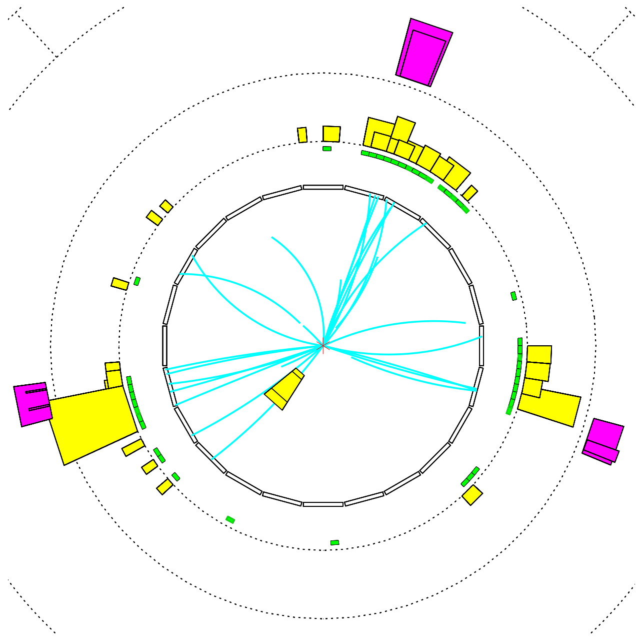
# Particle Physics



# QCD

**QUANTUM ELECTRODYNAMICS:** is the quantum theory of the electromagnetic interaction.

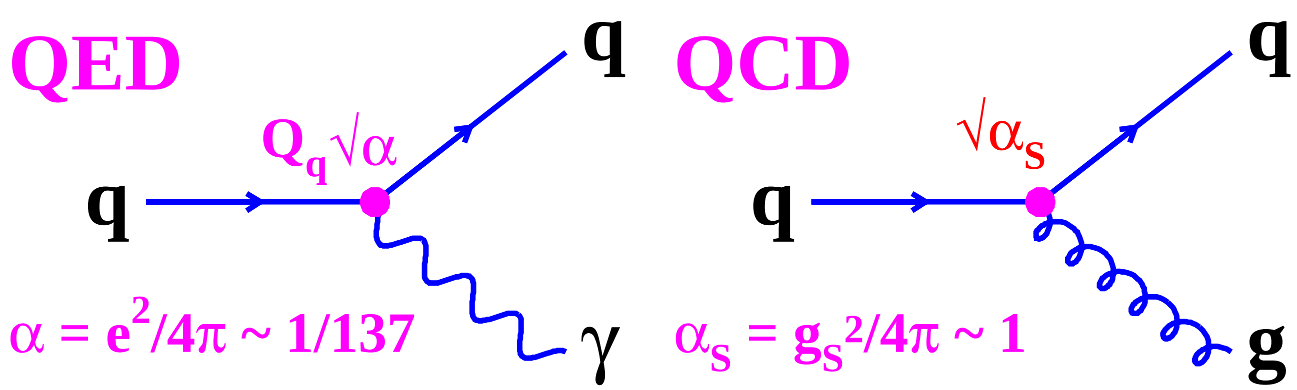
- ★ mediated by massless photons
- ★ photon couples to electric charge, $e$
- ★ Strength of interaction : $\langle\psi_f|\hat{\mathrm{H}}|\psi_i\rangle \propto \sqrt{\alpha}$.

$$\alpha = \frac{e^2}{4\pi}$$

**QUANTUM CHROMO-DYNAMICS:** is the quantum theory of the strong interaction.

- ★ mediated by massless gluons, *i.e.* $1/q^2$ propagator
- ★ gluon couples to "strong" charge
- ★ Only quarks have non-zero "strong" charge, therefore only quarks feel strong interaction

Basic QCD interaction looks like a stronger version of QED, $\alpha_S > \alpha_{EM}$




( subscript em is sometimes used to distinguish the $\alpha_{em}$ of electromagnetism from $\alpha_S$).

# COLOUR

## In QED:

- ★ Charge of QED is electric charge.
- ★ Electric charge - conserved quantum number.

## In QCD:

- ★ Charge of QCD is called "COLOUR"
- ★ COLOUR is a conserved quantum number with 3 VALUES labelled "red", "green" and "blue"

Quarks carry "COLOUR" $r \ g \ b$

Anti-quarks carry "ANTI-COLOUR" $\overline{r} \ \overline{g} \ \overline{b}$

Leptons, $\gamma$, $\mathrm{W}^{\pm}$, $\mathrm{Z}^0$ DO NOT carry colour, i.e. "have colour charge zero" $\longrightarrow$ DO NOT participate in STRONG interaction.

**Note:** Colour is just a label for states in a non-examinable SU(3) representation

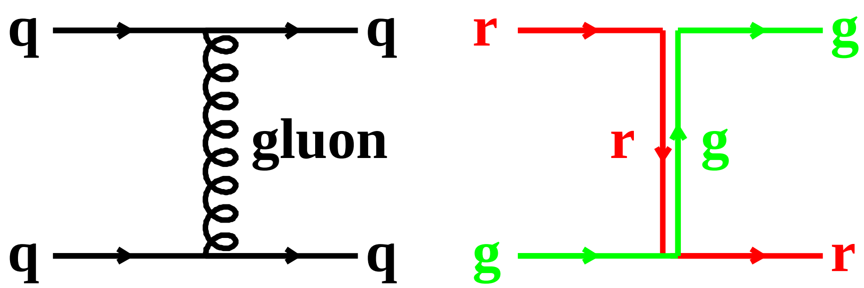
$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

# GLUONS

## In QCD:

★ Gluons are MASSLESS spin-1 bosons

Consider a red quark scattering off a green quark. Colour is exchanged but always conserved.




## UNLIKE QED:

★ Gluons carry the charge of the interaction.

★ Gluons come in different colours.

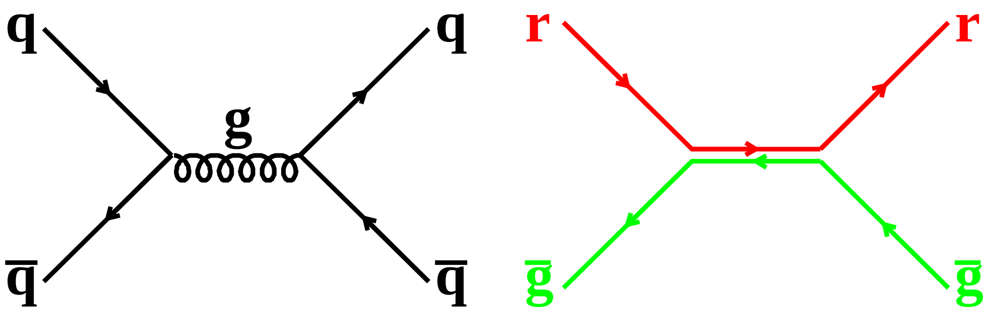
**Expect 9 gluons** (3 colours $\times$ 3 anti-colours)

$$r\overline{b},\ r\overline{g},\ g\overline{r},\ g\overline{b},\ b\overline{g},\ b\overline{r}$$
$$r\overline{r},\ g\overline{g},\ b\overline{b}$$

**However:** Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\overline{r} + g\overline{g} + b\overline{b})$ is colourless and does not take part in the strong interaction.

# Colour at Work

## EXAMPLE: $q\overline{q}$ Annihilation




Normally do not show colour on Feynman diagrams - colour is conserved.

## QED POTENTIAL:

$$V_{\rm QED} = -\frac{\alpha}{r}$$

## QCD POTENTIAL:

At short distances QCD potential looks similar

$$V_{\rm QCD} = -\frac{4}{3}\frac{\alpha_S}{r}$$

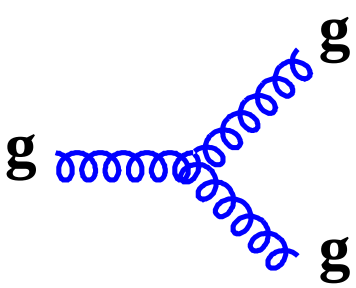
apart from $\frac{4}{3}$ colour factor.

Note: the colour factor (4/3) arises because more than one gluon can participate in the process $q \to qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states

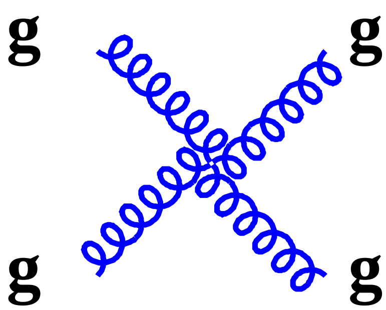
# SELF-INTERACTIONS

At this point, QCD looks like a stronger version of QED. This is true up to a point. However, in practice QCD behaves very differently to QED. The similarities arise from the fact that both involve the exchange of **MASSLESS** spin-1 bosons. The big difference is that **GLUONS** carry **colour** "charge".

## GLUONS CAN INTERACT WITH OTHER GLUONS:

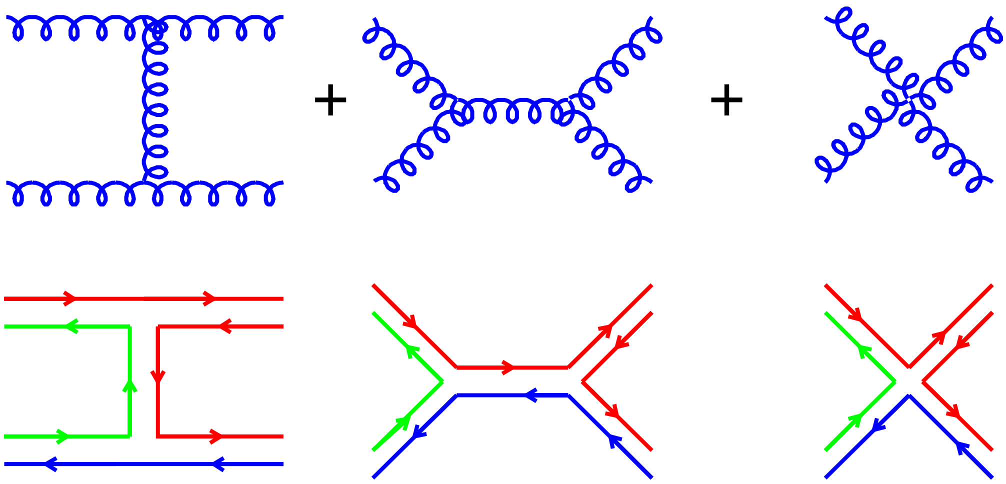



3 GLUON VERTEX




4 GLUON VERTEX

**EXAMPLE:** Gluon-Gluon Scattering $gg \to gg$



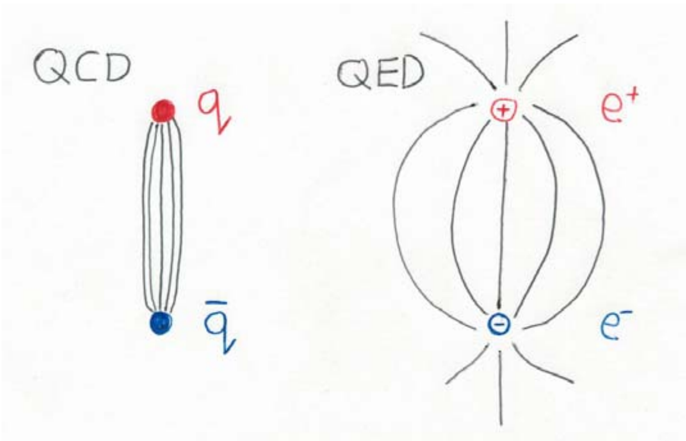

e.g. $r\bar{g} + g\bar{b} \to r\bar{r} + r\bar{b}$

# CONFINEMENT

<u>NEVER OBSERVE:</u> single FREE quarks/gluons

- ★ quarks are always confined within hadrons
- ★ This is a consequence of the strong self-interactions of gluons.

Qualitatively, picture the colour field between two quarks. The gluons mediating the force act as additional sources of the colour field - they attract each other. The gluon-gluon interaction pulls the lines of colour force into a narrow tube or STRING. In this model the string has a ‘tension’ and as the quarks separate the string stores potential energy.




Energy stored per unit length $\sim$ constant.

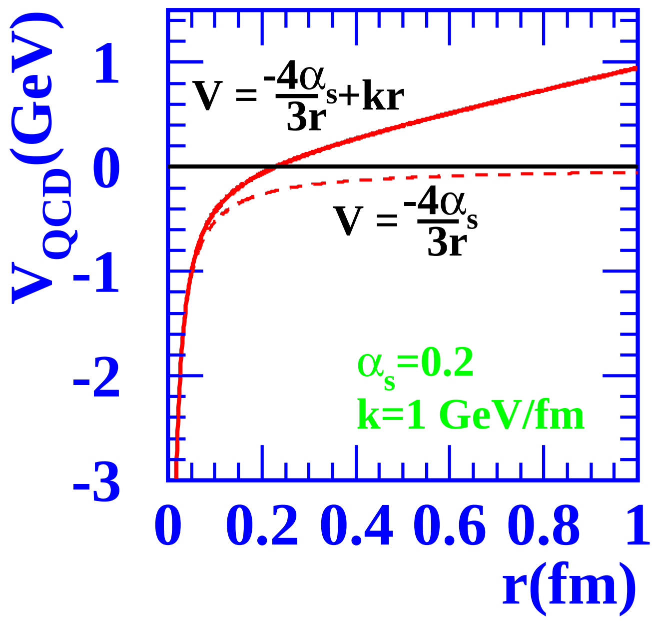
$$V(r) \propto r$$

★ Requires infinite energy to separate two quarks. Quarks always come in combinations with zero net colour charge: CONFINEMENT.

# How Strong is Strong ?

QCD Potential between quarks has two components:

- ★ "COULOMB"-LIKE TERM : $-\frac{4}{3}\frac{\alpha_S}{r}$
- ★ LINEAR TERM : $+kr$




Force between two quarks at separated by 10 m:

$$V_{QCD} = -\frac{\alpha_S}{r} + kr$$

with $k \approx 1\,\mathrm{GeV/fm}$

$$F = -\frac{dV}{dr} = \frac{\alpha_S}{r^2} + k$$

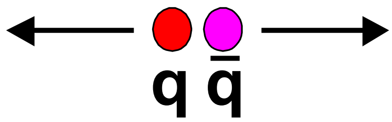
at large r $F = k = \frac{1.6 \times 10^{-10}}{10^{-15}} N$

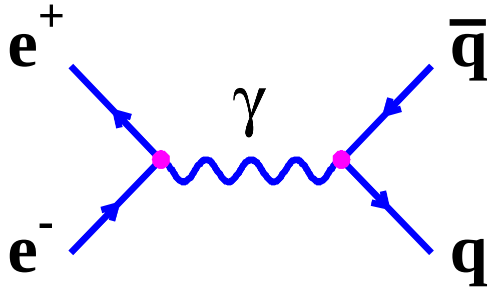
$$= 160000\ N$$

Equivalent to the weight of approximately 65 Widdecombes.

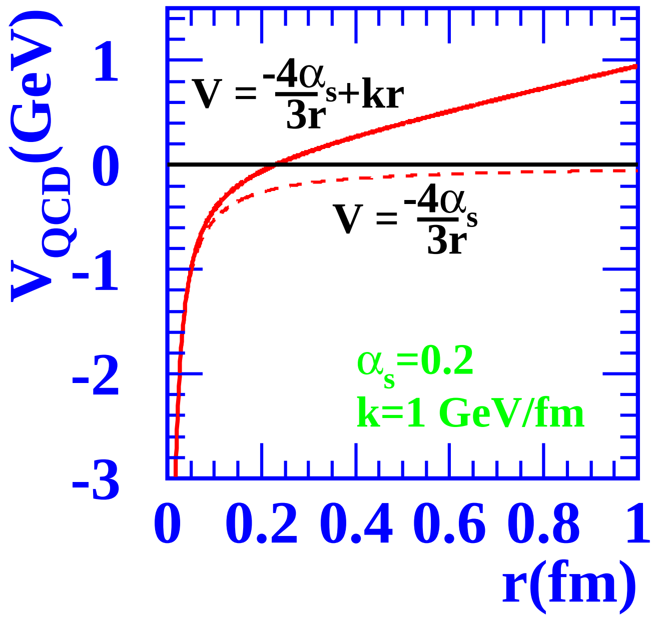
# JETS

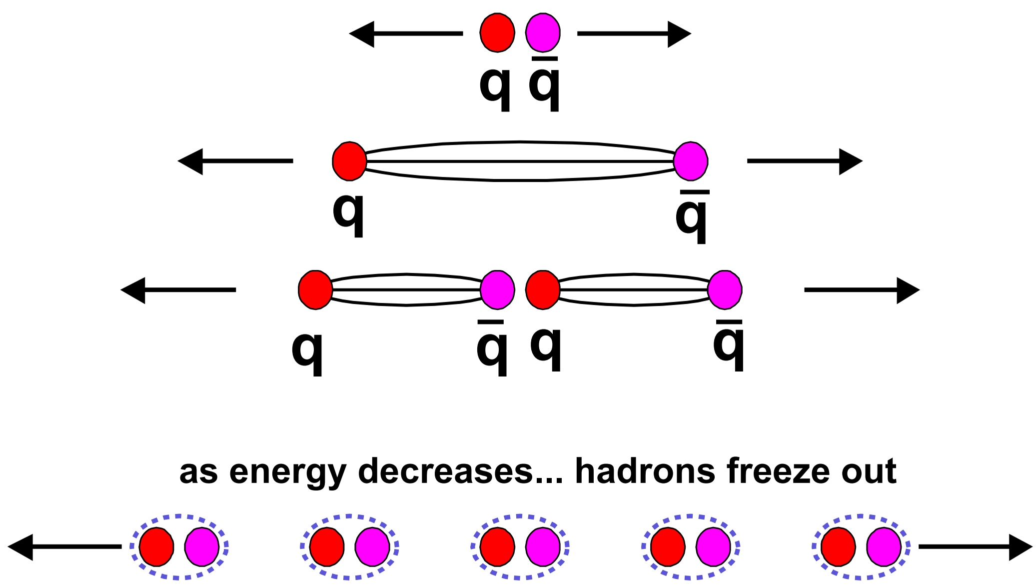
Consider the $q\bar{q}$ pair produced in $e^+e^- \to q\bar{q}$:



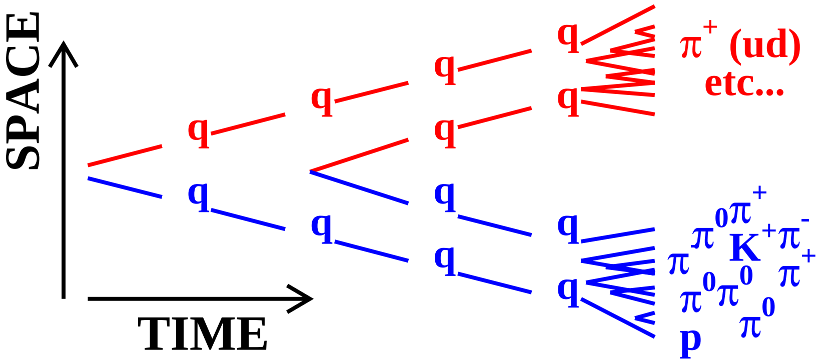



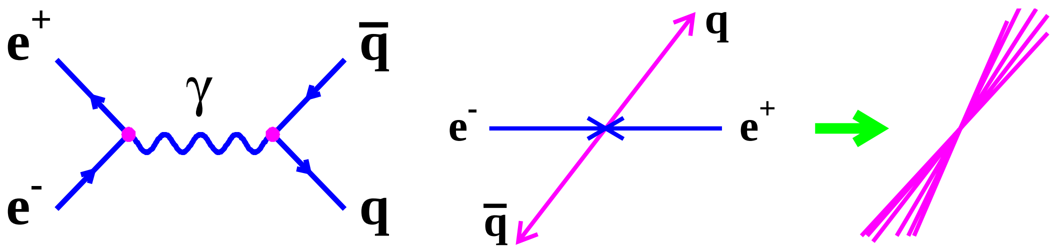

As the quarks separate, the energy stored in the colour field ('string') starts to increase linearly with separation. When $E_{stored} > 2m_q$ new $q\bar{q}$ pairs can be created.



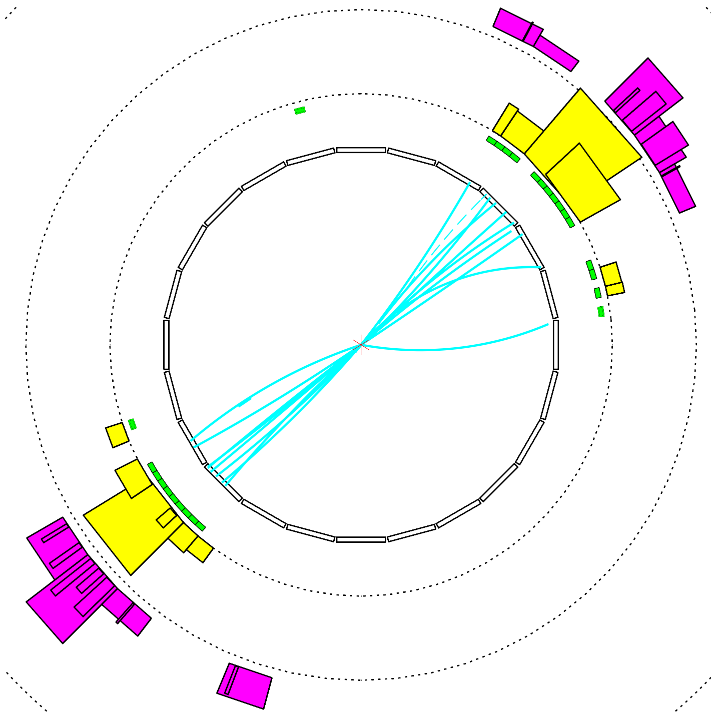



As quarks separate, more $q\bar{q}$ pairs are produced from the potential energy of the colour field. This process is called HADRONIZATION. Start out with quarks and end up with narrowly collimated JETS of HADRONS



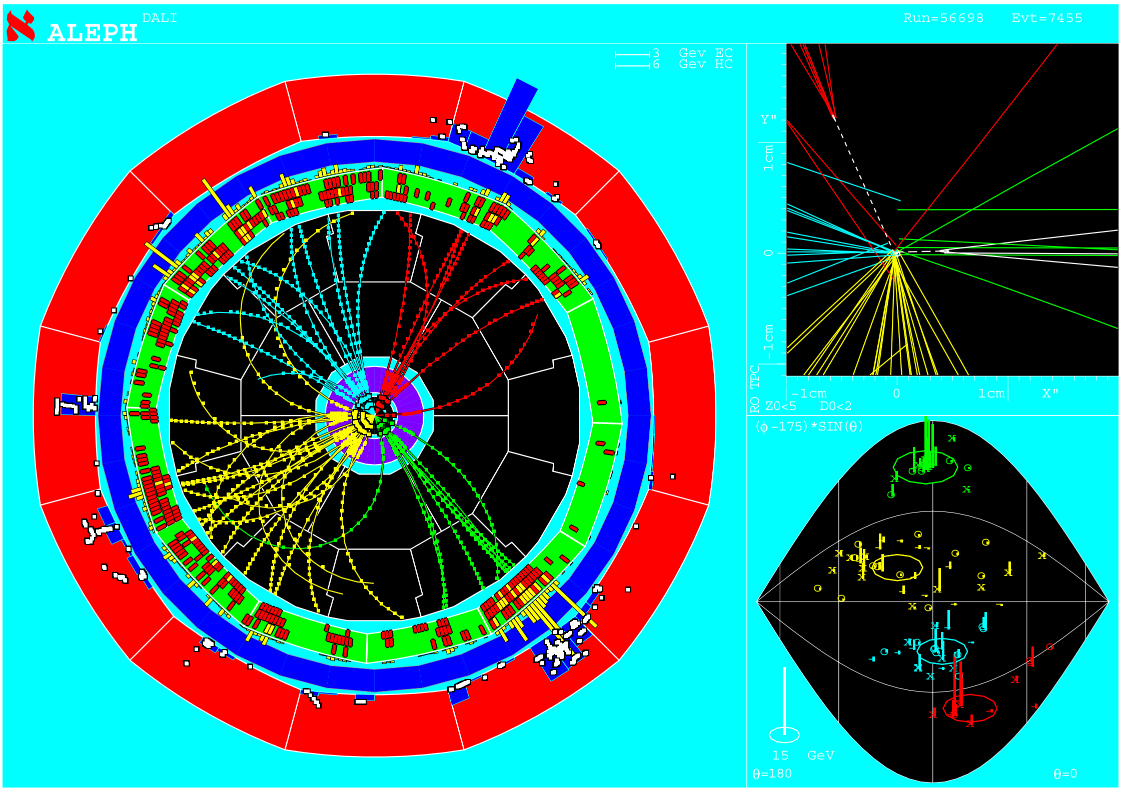




## Typical $e^+e^- \rightarrow q\bar{q}$ Event



The hadrons in a quark(anti-quark) jet follow the direction of the original quark(anti-quark). Consequently $e^+e^- \rightarrow q\bar{q}$ is observed as a pair of back-to-back jets of hadrons

# aside.....



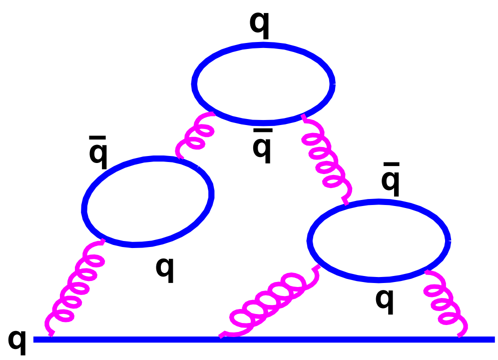

★ **You will now recognize the "Higgs" event from the cover of Handout I as**

$$e^+e^- \rightarrow \textbf{something} \rightarrow q\bar{q}q\bar{q}$$

# Running of $\alpha_S$

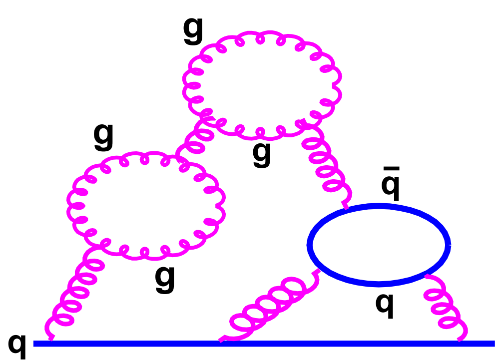
- ★ $\alpha_S$ specifies the strength of the strong interaction
- ★ BUT just as in QED, $\alpha_S$ isn't a constant, it "runs"
- ★ In QED the bare electron charge is screened by a cloud of virtual electron-positron pairs.
- ★ In QCD a similar effect occurs.

In QCD quantum fluctuations lead to a 'cloud' of virtual $q\bar{q}$ pairs




one of many (an infinite set) such diagrams analogous to those for QED.

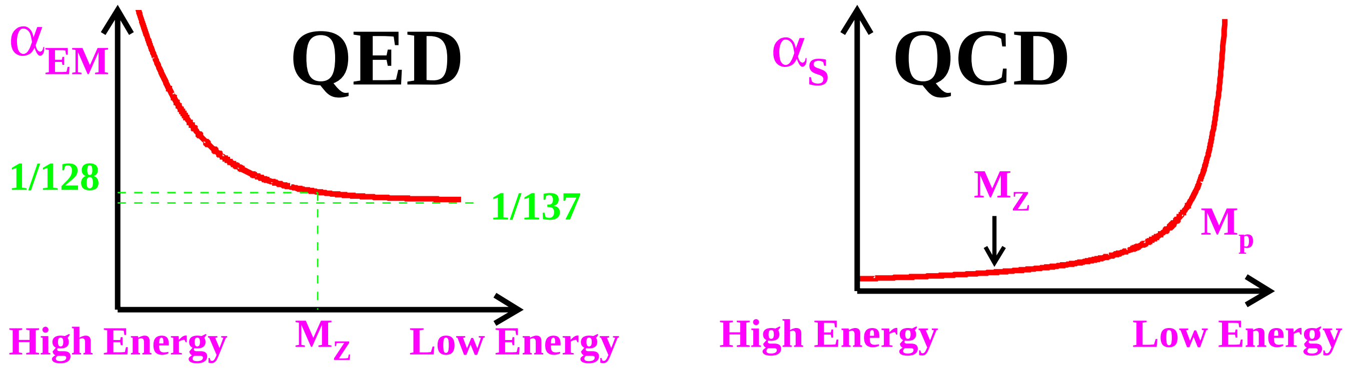
In QCD the gluon self-interactions ALSO lead to a 'cloud' of virtual gluons

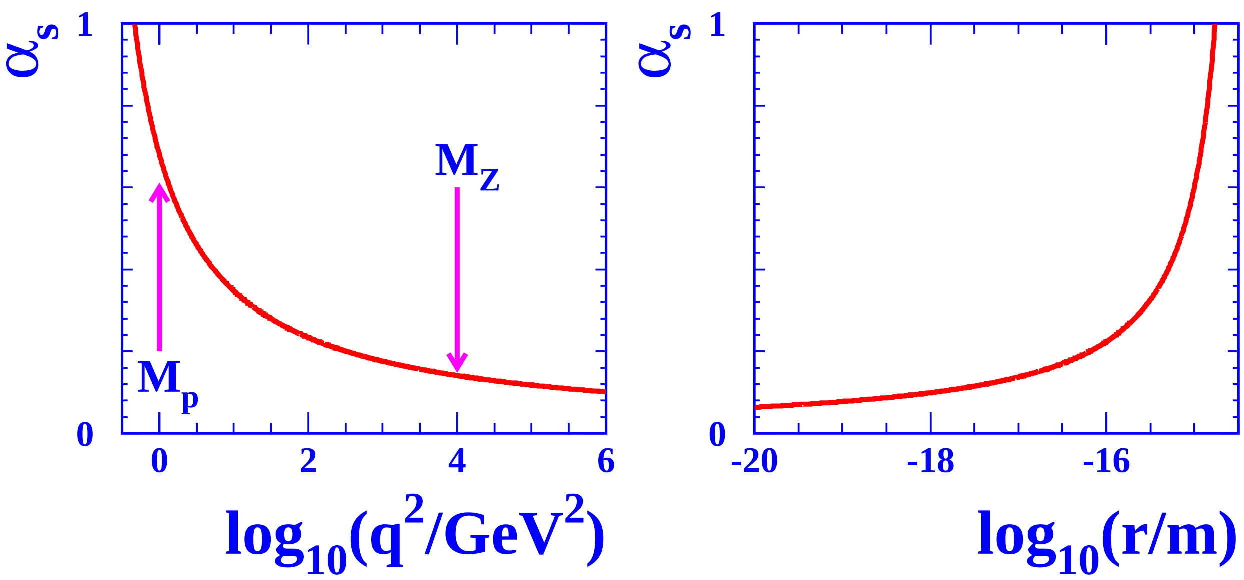



one of many (an infinite set) such diagrams. Here there is no analogy in QED, photons don't have self-interactions since they don't carry the charge of the interaction.

## Colour Anti-Screening

- ★ Due to the gluon self-interactions bare colour charge is screened by both virtual quarks and virtual gluons
- ★ The cloud of virtual gluons carries colour charge and the effective colour charge INCREASES with distance !
- ★ At low energies (large distances) $\alpha_S$ becomes large $\rightarrow$ can't use perturbation theory (not a weak perturbation)



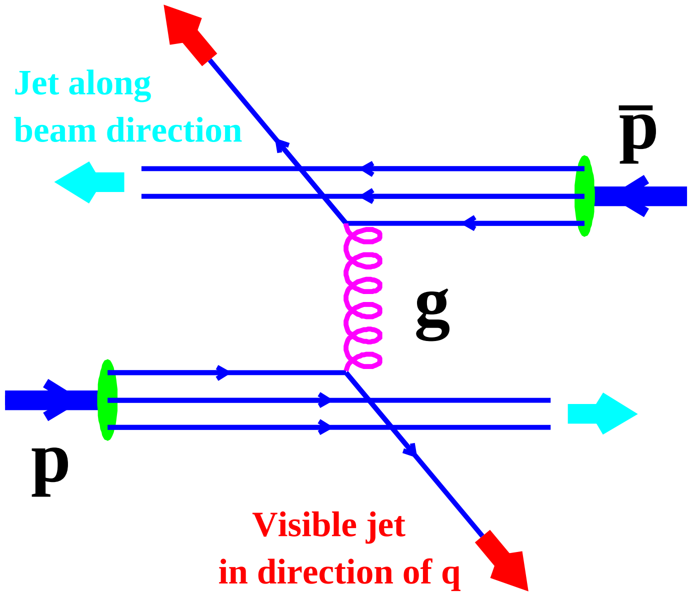



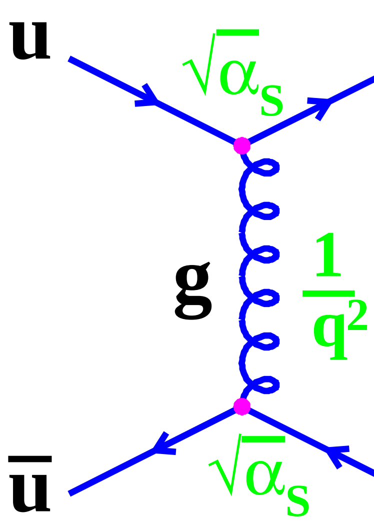

- ★ At High energies (short distances) $\alpha_S$ is small. In this regime treat quarks as free particles and can use perturbation theory

  ASYMPTOTIC FREEDOM
- ★ At $\sqrt{s} = 100\ \mathrm{GeV}, \alpha_S = 0.12$.

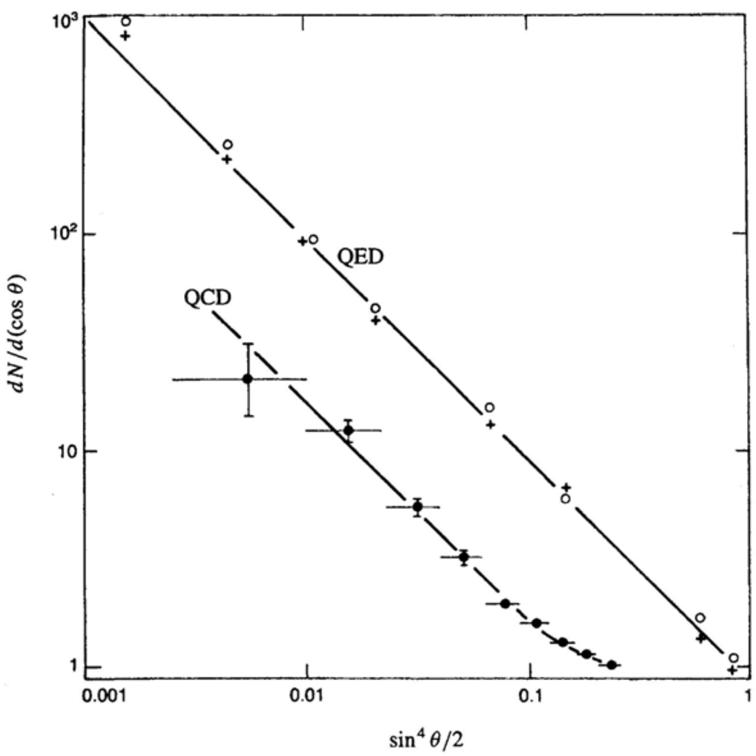
# Scattering in QCD

## EXAMPLE: High energy proton-antiproton scattering



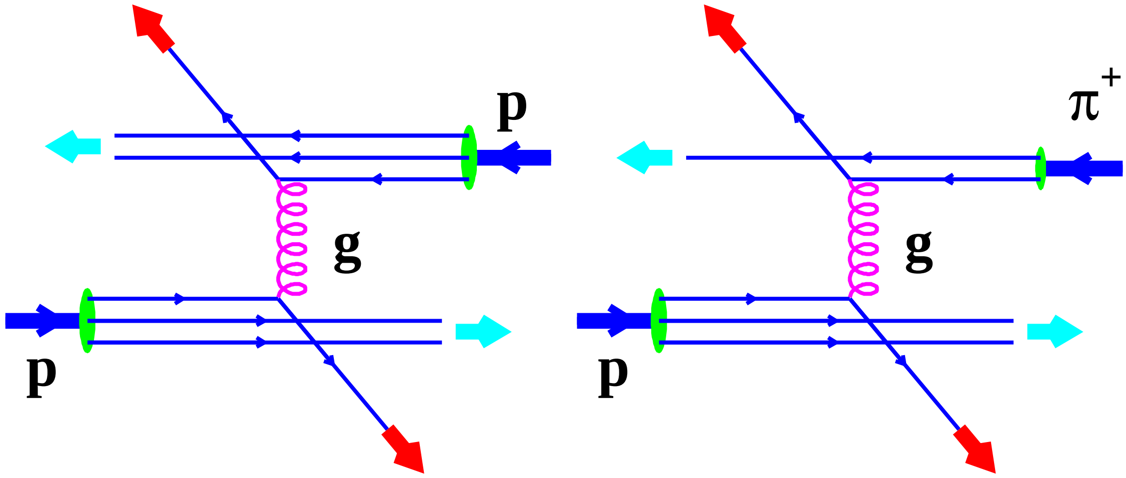




$$M \sim \frac{1}{q^2}\sqrt{\alpha_S}\sqrt{\alpha_S} \Rightarrow \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_S)^2}{\sin^4 \theta/2}$$



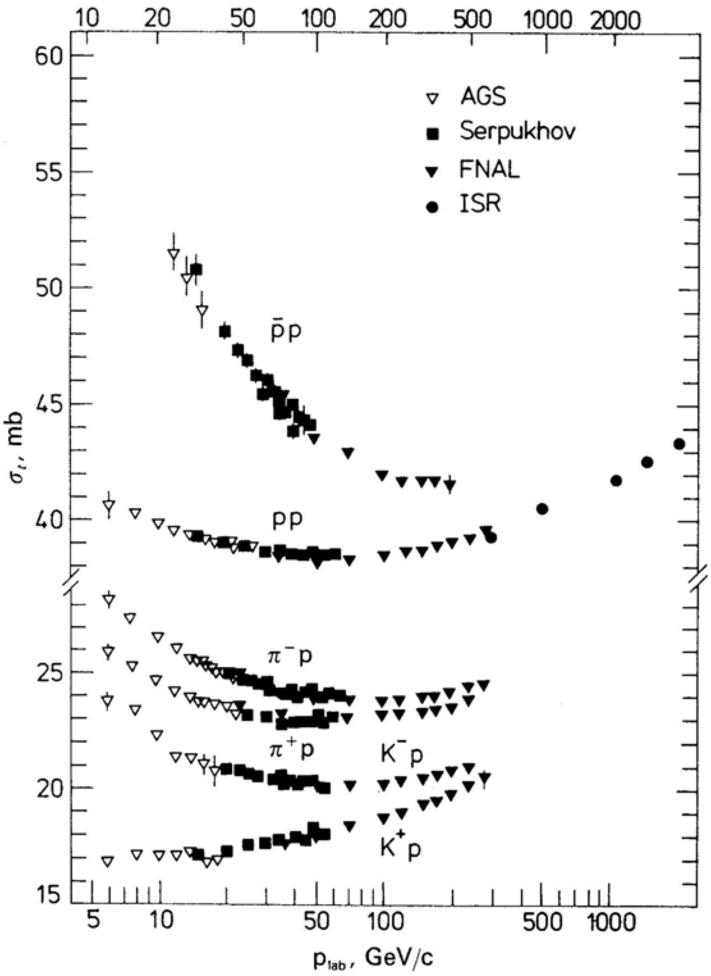

The upper points are the Geiger and Marsden data (1911) for the elastic scattering of $\alpha$ particles as they traverse thin gold and silver foils. The lower points show the angular distribution of the quark jets observed in proton-antiproton scattering at $q^2 = 2000\mathrm{GeV}^2$ Both follow the Rutherford formula for elastic scattering: $\sin^4 \frac{\theta}{2}$.

## EXAMPLE: $pp$ vs $\pi^+ p$ scattering




Calculate ratio of $\sigma(pp)_{\text{total}}$ to $\sigma(\pi^+ p)_{\text{total}}$

★ QCD does not distinguish between quark flavours, only COLOUR charge of quarks matters.




At high energy (E $\gg$ binding energy of quarks within hadrons) ratio of $pp$ and $p\pi$ total cross sections depends on number of possible quark-quark combinations.

**Predict**

$$\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2\times 3}{3\times 3} = \frac{2}{3}$$

**Experiment**

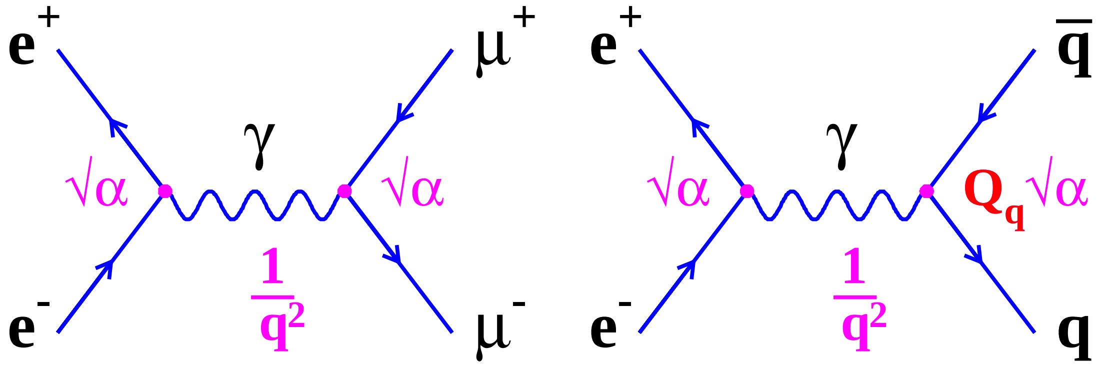
$$\frac{\sigma(\pi p)}{\sigma(pp)} \approx \frac{24\ mb}{38\ mb} \approx 0.63$$

# QCD in $e^+e^-$ Annihilation

Direct evidence for the existence of colour comes from $e^+e^-$ Annihilation.

★ Compare $e^+e^- \to \mu^+\mu^-, e^+e^- \to q\bar{q}$:

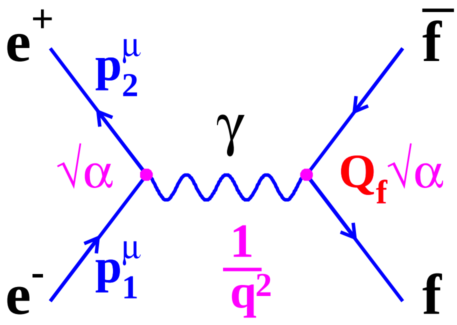
$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

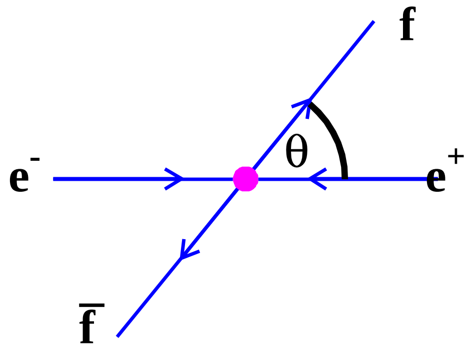



If we neglect the masses of the final state quarks/muons then the ONLY difference is the charge of the final state particles ($Q_\mu = -1, Q_q$ = $+\frac{2}{3}$ or $-\frac{1}{3}$ )

Start by calculating the cross section for the process $\sigma(e^+e^- \to f\bar{f})$. ($f\bar{f}$ represent a fermion-antifermion pair *e.g.* $\mu^+\mu^-$ or $q\bar{q}$).

see Handout II for the case where $f\bar{f} = \mu^+\mu^-$




**Electron/Positron beams along $z$-axis**

$$
\begin{aligned}
p_1^\mu &= (E, p_x, p_y, p_z) \\
p_1^\mu &= (E, 0, 0, E) \quad \text{neglecting } m_e \\
p_2^\mu &= (E, 0, 0, -E) \\
q^\mu &= p_1^\mu + p_2^\mu \\
&= (2E, 0, 0, 0) \\
q^2 &= 4E^2 = s
\end{aligned}
$$

**where $s$ is (centre-of-mass energy)$^2$.**

**Fermi's Golden rule and Born Approximation**

$$\frac{d\sigma}{d\Omega} = 2\pi |M|^2 \frac{d\rho(E_f)}{d\Omega}$$

**Matrix element $M$:**

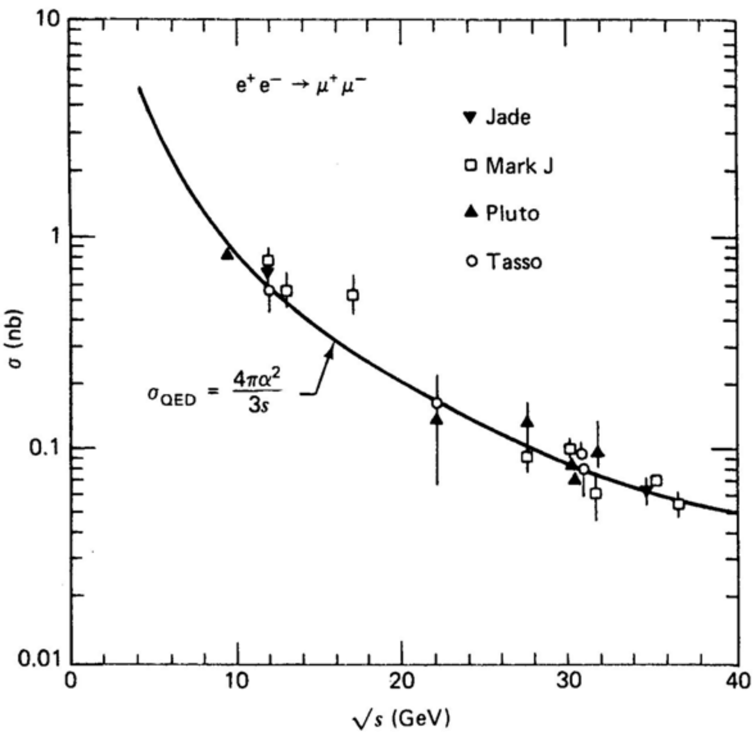
$$
\begin{aligned}
M &= \langle v_{e^+} | Q_e e | u_{e^-} \rangle \frac{1}{q^2} \langle v_{\bar{f}} | Q_f e | u_f \rangle \\
&= \frac{-4\pi\alpha Q_e Q_f}{q^2} \quad \text{with} \quad \alpha = \frac{e^2}{4\pi} \\
\frac{d\sigma}{d\Omega} &= 2\pi \frac{(-4\pi\alpha Q_e Q_f)^2}{q^4} \frac{E^2}{(2\pi)^2} \frac{1}{4} (1 + \cos^2\theta) \\
&= \frac{\alpha^2 Q_f^2}{4s} (1 + \cos^2\theta)
\end{aligned}
$$

★ $(1+\cos^2\theta)$ **comes from spin-1 photon "decaying" to two spin-half fermions.** see lecture on Dirac equation

**Total cross section for** $e^+e^- \to f\bar{f}$

$$\begin{aligned}
\sigma &= \int \frac{d\sigma}{d\Omega} d\Omega \\
&= \int_0^{2\pi}\int_0^{\pi} \frac{\alpha^2 Q_f^2}{4s}(1+\cos^2\theta)\sin\theta\, d\theta\, d\phi \\
&= \frac{\pi\alpha^2 Q_f^2}{2s}\int_{-1}^{+1}(1+y^2)\,dy \qquad (y=\cos\theta) \\
&= \frac{4\pi\alpha^2 Q_f^2}{3s}
\end{aligned}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$




$\sigma(e^+e^- \to \mu^+\mu^-)$ **for** $e^+e^-$ **collider data at centre-of-mass energies 8-36 GeV**

Back to

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

For a single quark flavour of a given colour

$$R = Q_q^2$$

However, we measure $e^+e^- \to \text{jets}$ not $e^+e^- \to u\bar{u}$. A jet from a u-quark looks just like a jet from a d-quark... Need to sum over flavours (u,d,c,s,t,b) and colours ($r$, $g$, $b$).

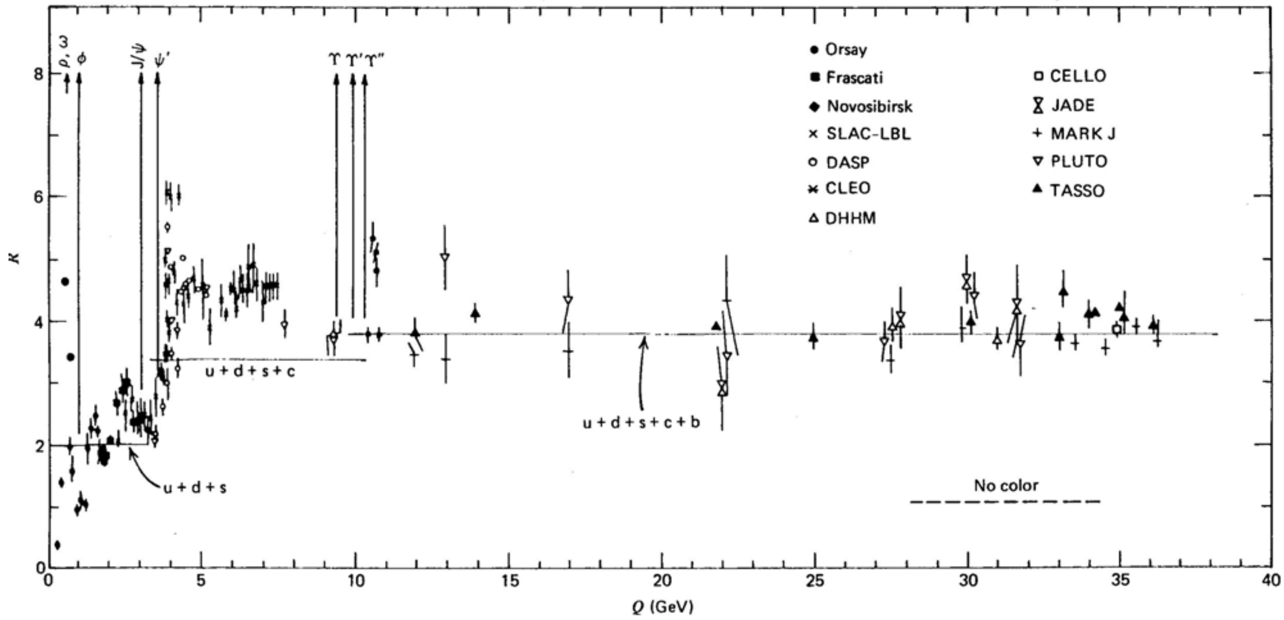
$$R = 3\sum_i Q_i^2 \qquad (3\text{colours})$$

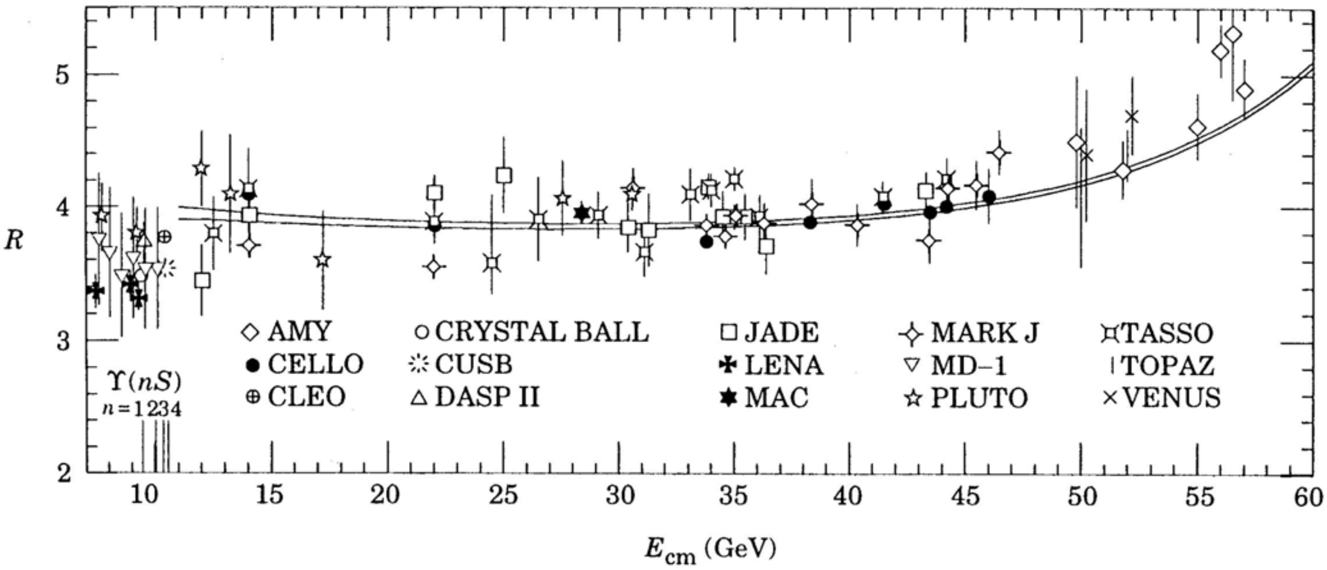
where the sum is over all quark flavours kinematically accessible at centre-of-mass energy, $\sqrt{s}$, of collider.

| Energy | Ratio R | |
|---|---|---|
| $\sqrt{s} > 2m_s$ ~1 GeV | $3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9})$ u,d,s | = $2$ |
| $\sqrt{s} > 2m_c$ ~4 GeV | $3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9})$ u,d,s,c | = $3\frac{1}{3}$ |
| $\sqrt{s} > 2m_b$ ~10 GeV | $3(..+\frac{1}{9})$ u,d,s,c,b | = $3\frac{2}{3}$ |
| $\sqrt{s} > 2m_t$ ~350 GeV | $3(..+\frac{4}{9})$ u,d,s,c,b,t | = $5$ |

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

**Data: $\sqrt{s}$ from $0 - 40$ GeV**




- ★ $R_\mu$ **increases in steps with** $\sqrt{s}$
- ★ $\sqrt{s} < 11\ GeV$ **region complicated by resonances: charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$).**
- ★ $R_\mu$ **Data exclude 'no colour' hypothesis.**

**STRONG EVIDENCE for COLOUR**

# Experimental Evidence for Colour

★ $R_\mu$

★ The existence of the $\Omega^-(sss)$

The $\Omega^-(sss)$ is a (L=0) spin-$\frac{3}{2}$ baryon consisting of 3 strange-quarks. The wave-function

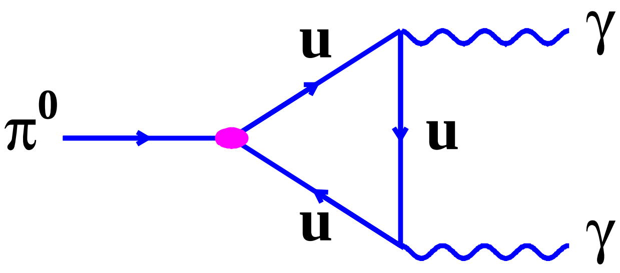
$$\psi = s\uparrow s\uparrow s\uparrow$$

is SYMMETRIC under particle interchange. However quarks are FERMIONS, therefore require an ANTI-SYMMETRIC wave-function, *i.e.* need another degree of freedom, namely COLOUR.

$$\begin{aligned} \psi &= (s\uparrow s\uparrow s\uparrow)\psi_{colour} \\ \psi_{colour} &= \frac{1}{\sqrt{6}}(rgb+gbr+brg-grb-rbg-bgr) \end{aligned}$$

★ $\pi^0 \rightarrow \gamma\gamma$ decay rate

Need colour to explain $\pi^0 \rightarrow \gamma\gamma$ decay rate.



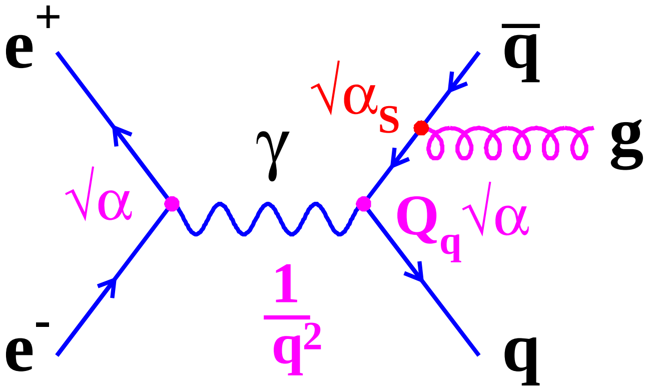

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) \quad \propto \quad N^2_{colour}$$

EXPT : $N_{colour} = 2.99 \pm 0.12$

# Evidence for Gluons

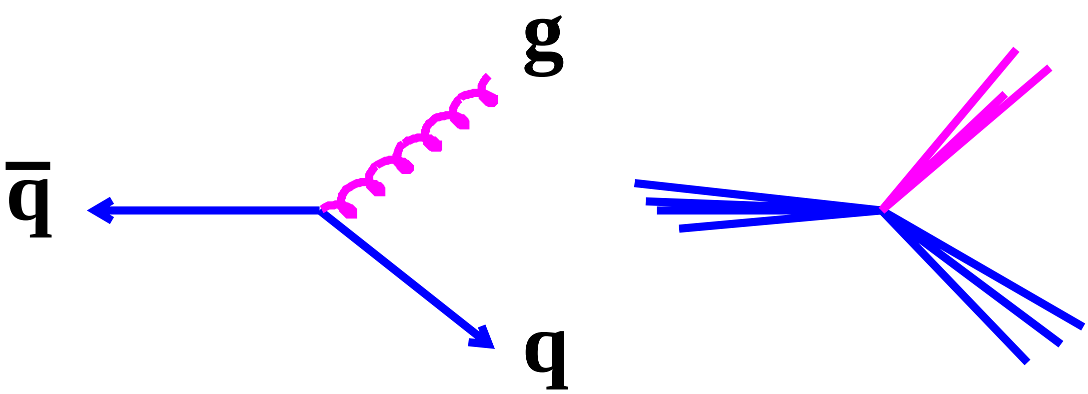
In QED, electrons can radiate photons. In QCD quarks can radiate gluons.

$e^+e^- \rightarrow q\bar{q}g$



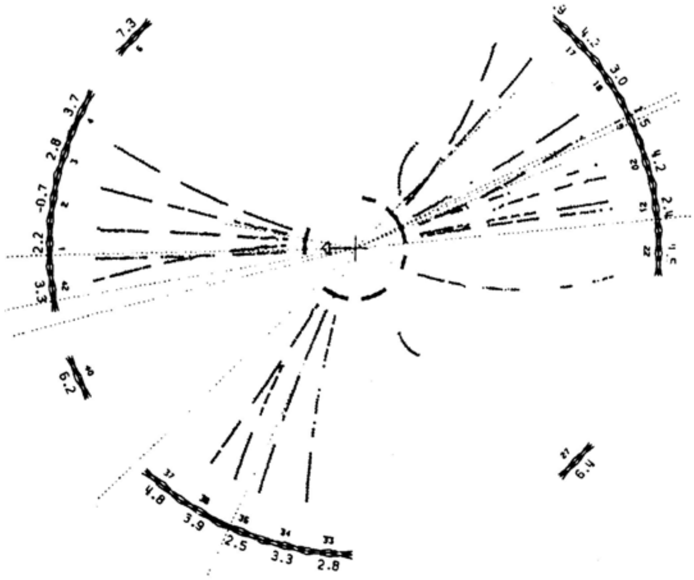

giving an extra factor of $\sqrt{\alpha_S}$ in the matrix element, i.e. an extra factor of $\alpha_S$ in cross section.

In QED we can detect the photons. In QCD we never see free gluons due to confinement.
Experimentally detect gluons as an additional jet: 3-Jet Events.

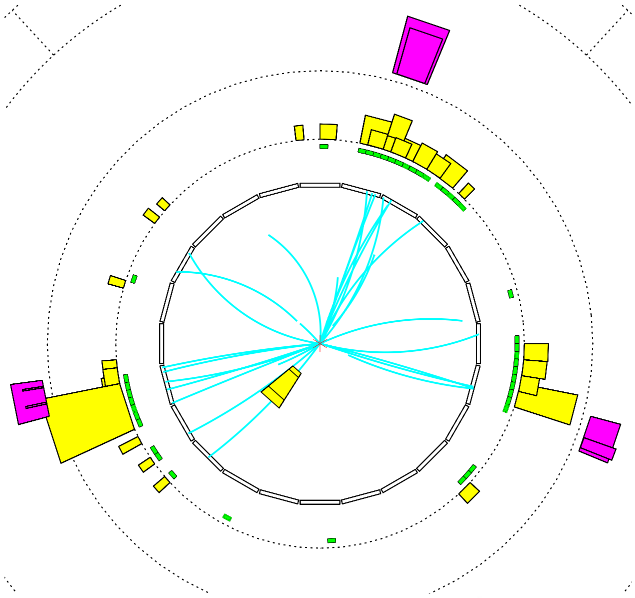



★ Angular distribution of gluon jet depends on gluon spin

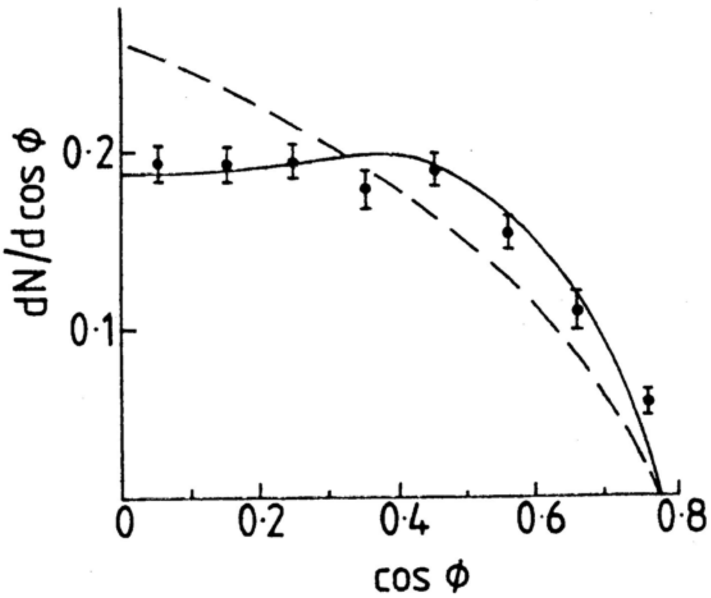
# 3-Jet Events and Gluon Spin



JADE $\sqrt{s} = 31$ GeV
Direct Evidence for Gluons (1978)



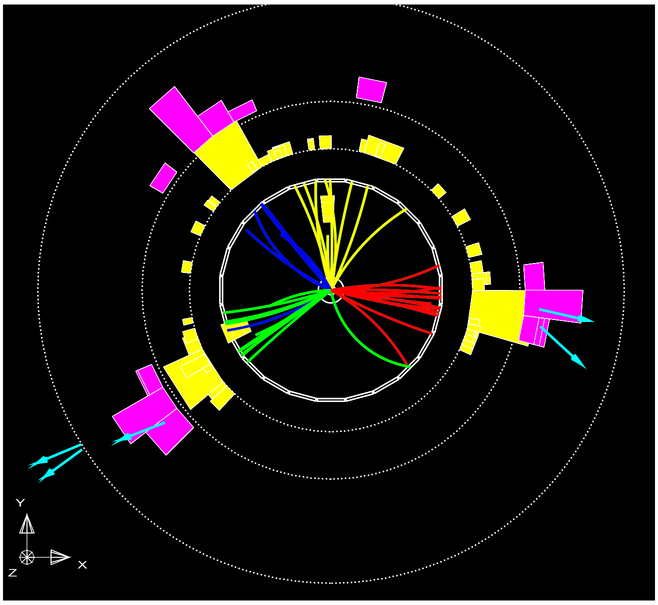
OPAL $\sqrt{s} = 91$ GeV
(1990)



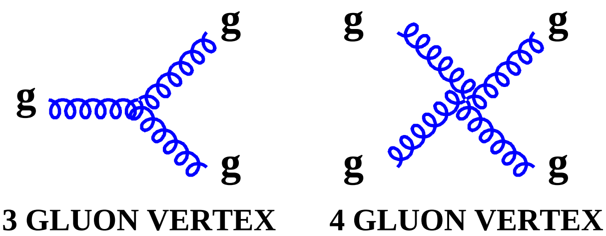

Distribution of the angle, $\phi$ between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cms frame). $\phi$ depends on the spin of the gluon.

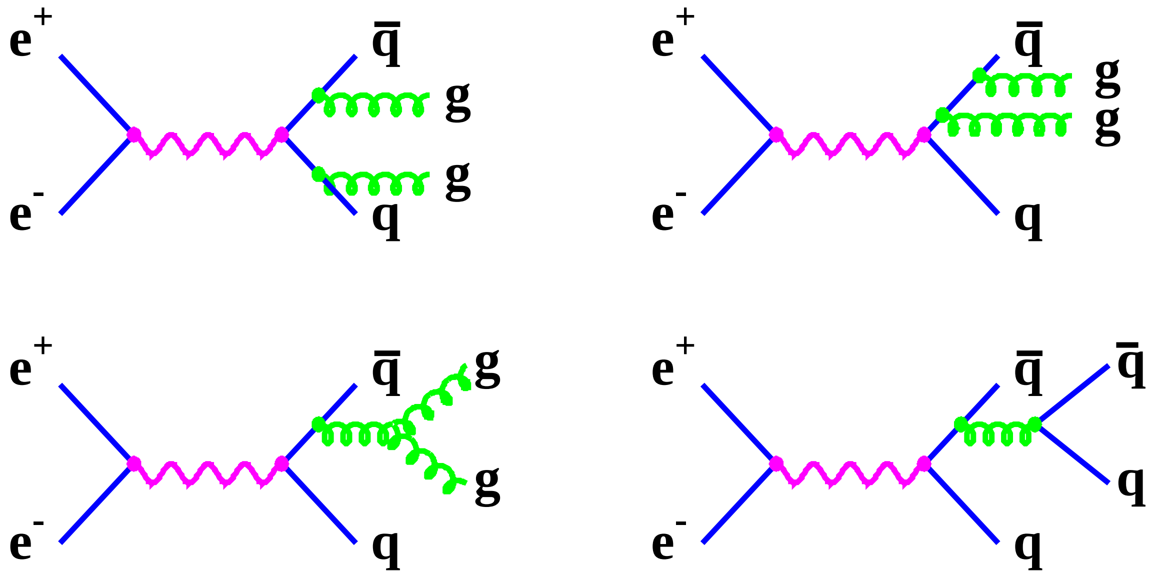
$\Rightarrow$ GLUON is SPIN-1

# Gluon Self-Interactions




Direct Evidence for the existence of the the gluon self-interactions from 4-JET events.



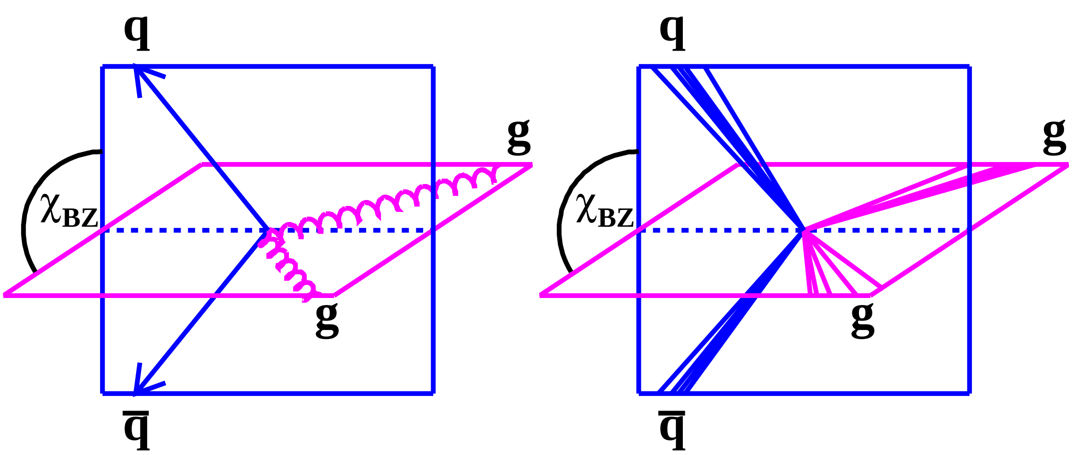

★ Angular distribution of jets is sensitive to existence triple gluon vertex:

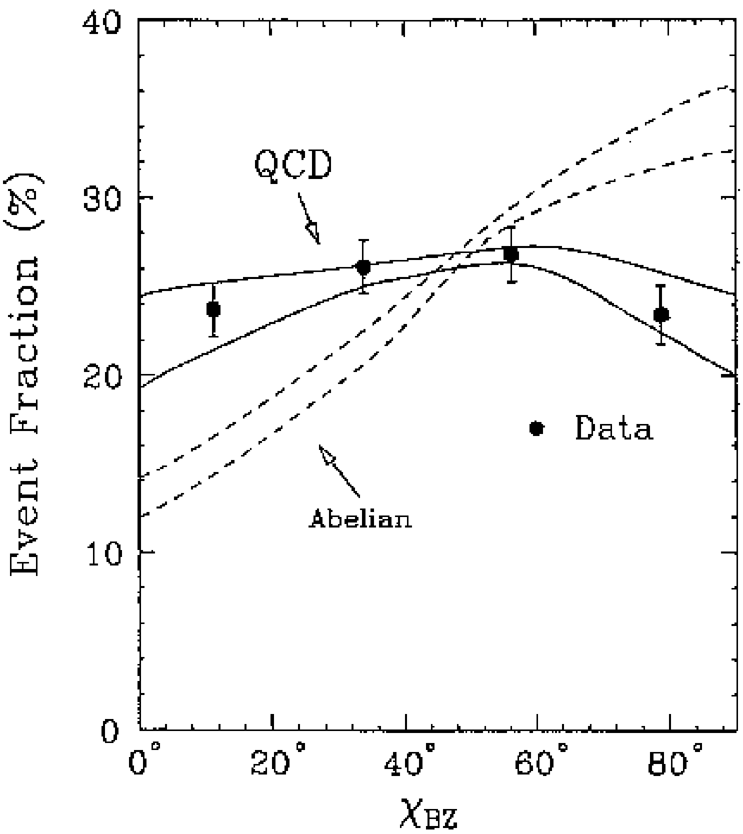
- ★ $q\bar{q}g$ vertex consists of 2 spin-$\frac{1}{2}$ quarks and a spin-1 gluon.
- ★ $ggg$ vertex consists of 3 spin-1 gluons, $\therefore$ different angular distribution.

## Experimentally:

★ Define the two lowest energy jets as the gluons. (gluon jets are more likely to be low energy than quark jets)

★ Measure angle between the plane containing the 'quark' jets and the plane containing the 'gluon' jets, $\chi_{\mathrm{BZ}}$

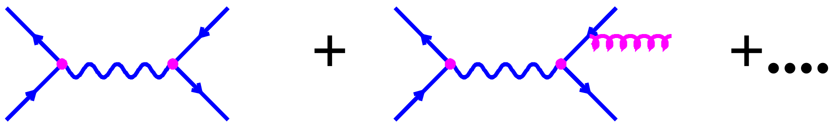



Gluon self-interactions are required to describe the experimental data. Theory without self-interactions (ABELIAN) is inconsistent with observations

# Measuring $\alpha_S$

$\alpha_S$ can be measured in many ways. The cleanest is from $R_\mu$: In practice measure



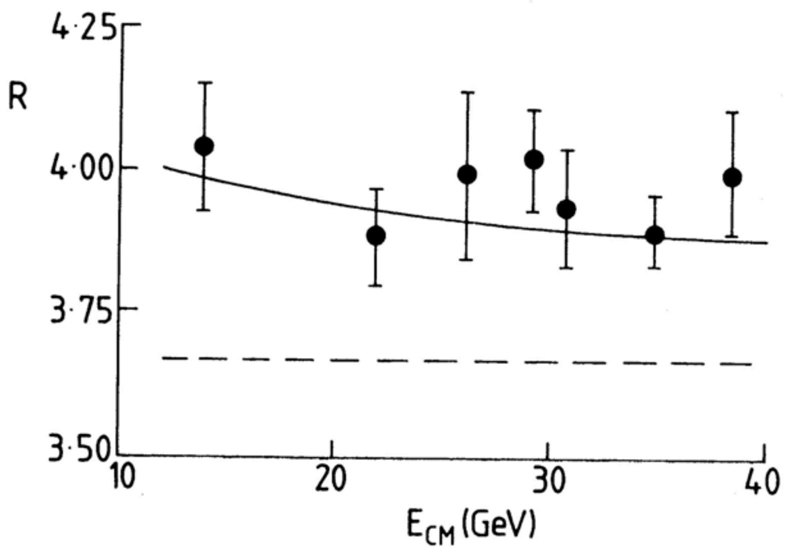
*i.e.* don't distinguish 2/3 jets. So measure

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\text{not} \quad R_\mu = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

When gluon radiation is included :

$$R_\mu = 3\sum Q_q^2\left(1 + \frac{\alpha_S}{\pi}\right)$$



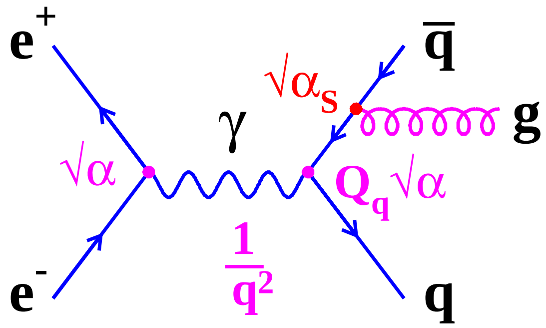

$$\sum_q Q_q^2 = 3\tfrac{2}{3}$$

Therefore $(1 + \frac{\alpha_S}{\pi}) \approx \frac{3.9}{3.66}$

giving $\quad \alpha_S(q^2 = 25^2) \approx 0.20$

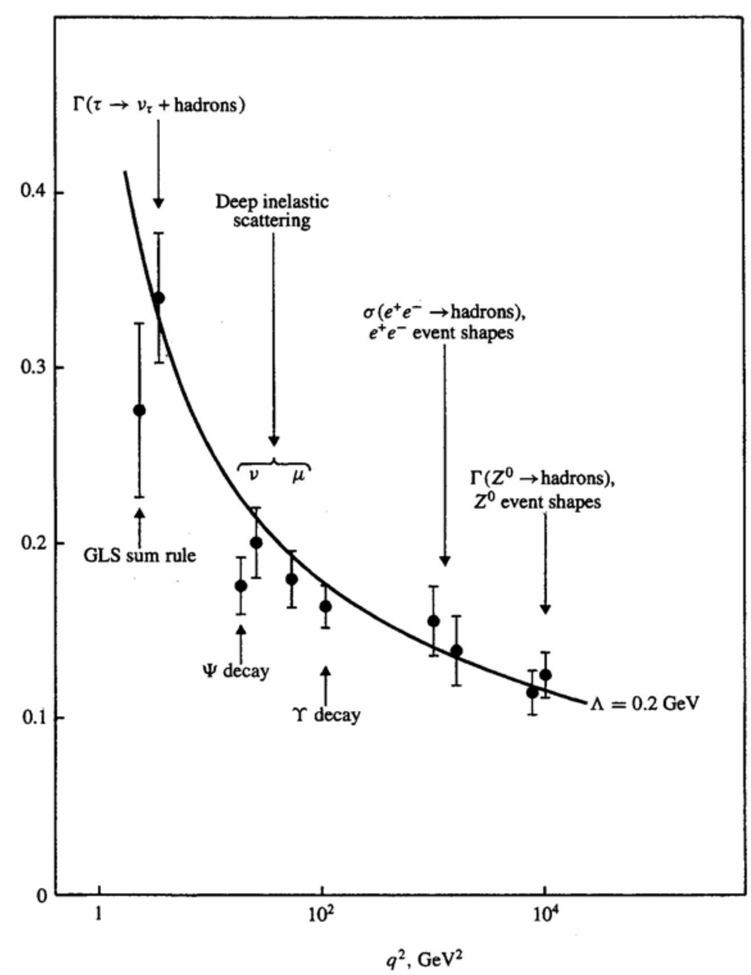
## Many other ways to measure $\alpha_S$ ....

**e.g. 3 jet rate : $e^+e^- \rightarrow q\bar{q}g$**




$$\frac{\sigma(3\text{ jet})}{\sigma(2\text{ jet})} = \frac{\sigma(q\bar{q}g)}{\sigma(q\bar{q})} \propto \alpha_S$$

# $\alpha_S$ Summary




**Summary of $\alpha_S$ measurements**

**$\alpha_S$ RUNS !**

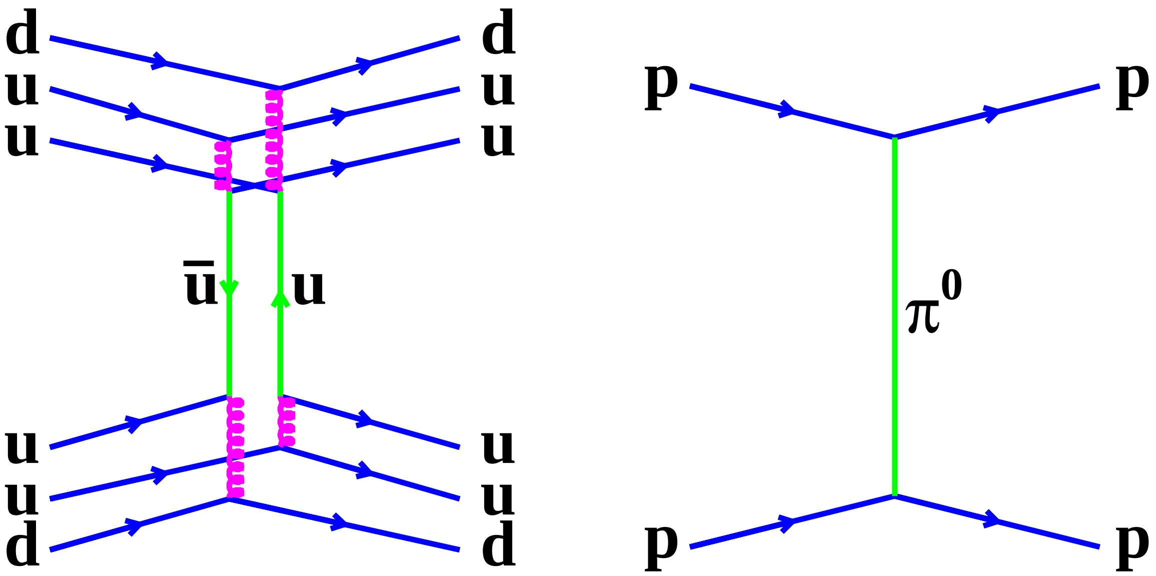
# Nucleon-Nucleon Interactions

★ Bound $qqq$ states (e.g. Protons and Neutrons) are COLOURLESS (COLOUR SINGLETS).

★ They can only interact via COLOURLESS intermediate states - i.e. not by single gluons. (conservation of colour charge)

★ Interact by exchange of PIONS

★ One possible diagram shown below :




★ Nuclear potential is YUKAWA potential with

$$V(\tilde{r}) = -\frac{g^2}{4\pi r} e^{-m_\pi r}$$

★ Short range force : range $\sim (m_\pi)^{-1}$

$$\begin{aligned} \text{Range} \quad R &= (0.140\ \text{GeV})^{-1} \\ &= 7\ \text{GeV}^{-1} \\ &= 7\hbar c/(1.6 \times 10^{-10})\ m \\ &= 1.4\ fm \end{aligned}$$