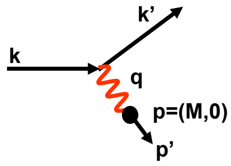
3.6

Kinematics of inelastic scattering on proton at rest



$$q = k - k'$$
$$W^2 = p'^2 = (p+q)^2 = p^2 + 2p\cdot q + q^2 = M^2 + 2M\nu + q^2$$

W – invariant mass of final state
(has to contain at least one baryon)
$W^2 = M^2$ elastic collision

$$W^2 - M^2 = 2M\nu - Q^2 > 0$$

$$Q^2 = 4EE'\sin^2\frac{\theta}{2} = -q^2$$

$$\nu = \frac{p\cdot q}{M} = E - E'$$

ν – energy lost by the incoming electron.

$$x = -\frac{q^2}{2p\cdot q}$$

x – 'Bjorken x',
0<x<1 inelastic collision
x=1 elastic collision

$$W^2 - M^2 = 2p\cdot q - Q^2 > 0$$
$$\Rightarrow 2p\cdot q > Q^2$$
$$\Rightarrow \frac{Q^2}{2p\cdot q} < 1$$

$$y = \frac{p\cdot q}{k\cdot p}$$

y – fractional energy loss of incoming particle

$$y = \frac{p\cdot q}{k\cdot p} = \frac{M(E-E')}{EM} = 1 - \frac{E'}{E}$$

Variables are not independent. Kinematics can be described by any 2 of the above (except for ν and y).

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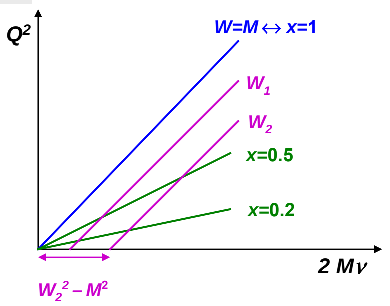
Bjorken's scaling variable x

$$x \equiv \frac{Q^2}{2M\nu} = \frac{Q^2}{2p\cdot q}$$

$$W^2 - M^2 = 2M\nu - Q^2$$
$$Q^2 = 2M\nu - (W^2 - M^2) =: 2M\nu x \quad \longleftarrow \text{"scaling"}$$
$$x = 1 - \frac{W^2 - M^2}{2M\nu}; \qquad 0 \le x \le 1$$

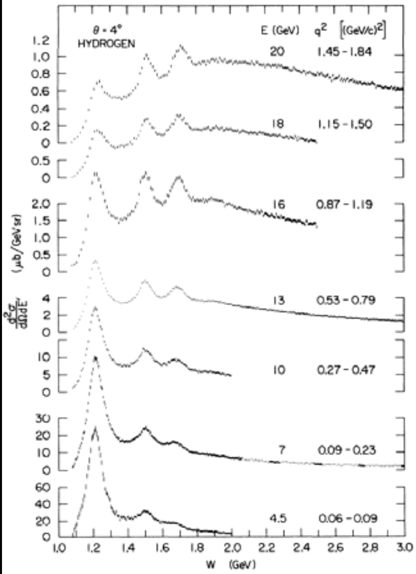


Kinematic relations

$$y \equiv \frac{\nu}{E} = \frac{p\cdot q}{ME} = \frac{2p\cdot q}{s}$$
$$xys = \frac{Q^2}{2p\cdot q}\frac{2p\cdot q}{s}s = Q^2$$

3.6

Inelastic ep – scattering



S.Stein et al., (SLAC E61) PRD 12, 1884 (1975)

2 independent quantities require to independent measurements:

→ observables: (E',θ)

For interpretation derive: **(Q^2,ν)** **(Q^2,W)** **(Q^2,x)** **(x,y)**

Characteristic feature:

W<2GeV resonance excitation
W>2GeV structureless 'deep inelastic' – scattering (with many particle final states)

(Unexpected) observation:
Deep inelastic cross section drops off much less than elastic one.

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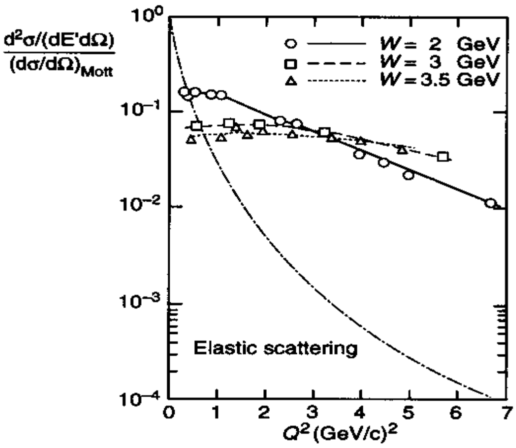
Deep inelastic ep – scattering



M. Breidenbach et al., PRL 23 (1969) 935

- *Elastic Q^2* – dependence

$$\frac{d\sigma / dE' d\Omega}{(d\sigma / d\Omega)_{Mott}} \cong \left(\frac{1}{(1+Q^2/0.71)^2} \right)^2 \propto Q^{-8}$$

- in deep – inelastic regime:

very weak Q^2 – dependence

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Inelastic cross section formula

Differential cross section for elastic electron scattering on spin ½ - particles:

$$\tau \equiv \frac{Q^2}{4M^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left(\underbrace{\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}}_{f_2(Q^2)}\cos^2\frac{\theta}{2}+\underbrace{2\tau G_M^2(Q^2)}_{f_1(Q^2)}\sin^2\frac{\theta}{2}\right)$$ **Lab frame**

$$\downarrow$$

$$\left(\frac{d\sigma}{dQ^2}\right)_{ep} = \frac{4\pi\alpha^2}{Q^4}\left(f_2(Q^2)\left(1-y-\frac{M^2y^2}{Q^2}\right)+\frac{1}{2}y^2 f_1(Q^2)\right)$$ **invariant**

Double differential cross section for inelastic electron scattering on spin ½ - particles:

$$\frac{d^2\sigma}{dQ^2dE'} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{\nu}\left[F_2(x)\cos^2\frac{\theta}{2}+\frac{2\nu}{M}F_1(x)\sin^2\frac{\theta}{2}\right]$$ **Lab frame**

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left(\frac{1}{x}F_2(x,Q^2)\left(1-y-\frac{M^2y^2}{Q^2}\right)+y^2F_1(x,Q^2)\right)$$

$$\downarrow Q^2 >> M^2y^2$$

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left(\frac{F_2(x,Q^2)}{x}(1-y)+y^2F_1(x,Q^2)\right)$$ **invariant**

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Bjorken scaling hypothesis (1967)

If scattering is caused by pointlike constituents (partons) structure functions must be independent of Q^2.

Compare elastic to inelastic cross section:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(f_2(Q^2)\left(1-y-\frac{M^2y^2}{Q^2}\right)+\frac{1}{2}y^2 f_1(Q^2)\right)$$

$$\downarrow \text{pointlike}: \quad f_i(Q^2)=1$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(\left(1-y-\frac{M^2y^2}{Q^2}\right)+\frac{1}{2}y^2\right)$$

=> $F_{1/2}$ should not have any Q^2 dependence.

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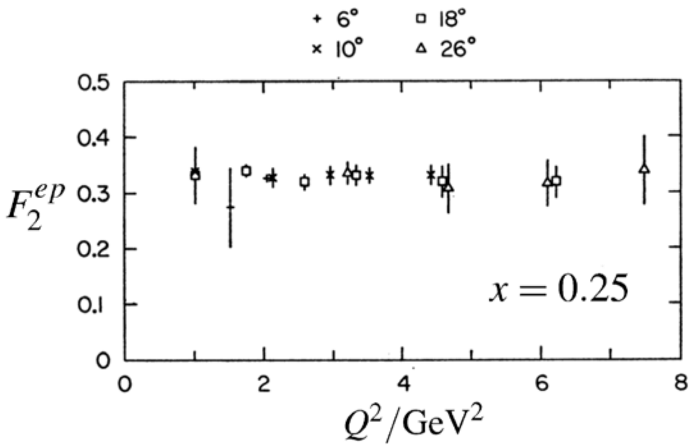
Measurement of structure function F_2



J.T. Friedman, H.W. Kendall,
Ann. Rev. Nucl. Sci. 22, 203 (1972)

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Parton spin

Compare:
Elastic scattering of electrons on spin ½ particle of mass M

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{eX}}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = 1 + \frac{Q^2}{2M^2}\tan^2\frac{\theta}{2}$$

Cross section parametrisation with structure functions F_1 and F_2

$$\frac{\frac{d^2\sigma}{d\Omega dE'}}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\nu}\left[F_2(x) + \frac{2\nu}{M}F_1(x)\tan^2\frac{\theta}{2}\right]$$

$$= \frac{F_2(x)}{\nu}\left[1 + \frac{2\nu}{M}\frac{F_1(x)}{F_2(x)}\tan^2\frac{\theta}{2}\right]$$

$$= \frac{F_2(x)}{\nu}\left[1 + \frac{2}{M}\frac{Q^2}{2Mx}\frac{x}{x}\frac{F_1(x)}{F_2(x)}\tan^2\frac{\theta}{2}\right]$$

$$\downarrow \mu \equiv Mx$$

$$= \frac{F_2(x)}{\nu}\left[1 + \frac{Q^2}{2\mu^2}\frac{2xF_1(x)}{F_2(x)}\tan^2\frac{\theta}{2}\right]$$

$$x \equiv \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu}$$
$$\Downarrow$$
$$\nu = \frac{Q^2}{2Mx}$$

Scattering on spin ½ particles: $F_2(x) = 2xF_1(x)$ **Callan – Gross – relation**

Scattering on spin 0 particles: $F_1(x)=0$

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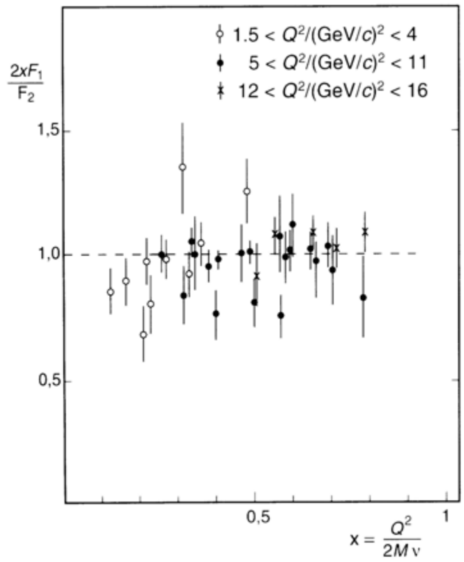
Callan – Gross – Relation

Data:



Scattering from spin ½ particles:

$$F_2(x) = 2xF_1(x)$$

Scattering from spin 0 particles:

$F_1(x)=0$

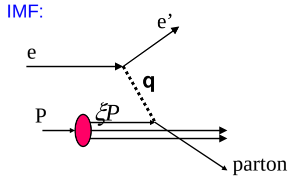
⇩

Partons have spin ½.

3.6

Quark – Parton – Model

IMF:



IMF – "Infinite Momentum Frame"
transvers momenta and masses can be neglected

$$p_{quark} = \xi P$$

$$p_{quark}^2 = p'^2_{quark} = (p_{quark} + q)^2$$

$$= p_{quark}^2 + 2p_{quark} \cdot q + q^2$$

$$= p_{quark}^2 + 2\xi P \cdot q - Q^2$$

$$\Downarrow$$

$$2\xi P \cdot q = Q^2$$

$$\Downarrow$$

$$\xi = \frac{Q^2}{2P \cdot q} \equiv x$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M\nu}$$

Cross section for electron – quark scattering:

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}e_i^2\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right]$$

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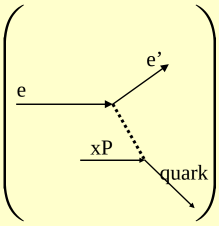
Cross section in Quark – Parton – Model

Cross section for electron proton = incoherent sum of electron – quark scattering:

$$\sigma_{ep} = \sum_i q_i(x)\sigma_i$$



quark density: probability to find quark i in momentum interval [x,x+dx]

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\sum_i \int_0^1 d\xi e_i^2 q_i(\xi)\delta(x-\xi)\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right]$$

Structure functions:

$$F_2(x) = x\sum_i \int_0^1 d\xi e_i^2 q_i(\xi)\delta(x-\xi) = x\sum_i e_i^2 q_i(x)$$

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

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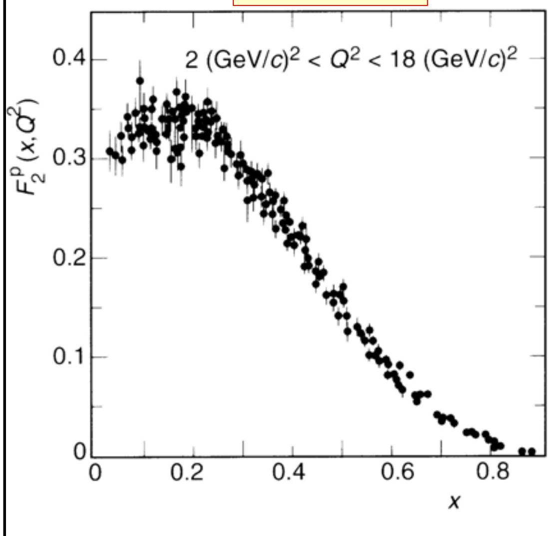
Parton distribution function F_2

$$F_2(x) = x\sum_i e_i^2 q_i(x)$$



status 1982

From double differential cross section measurements

$$\frac{d^2\sigma}{dQ^2dE'} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{\nu}\left[F_2(x)\cos^2\frac{\theta}{2} + \frac{2\nu}{M}F_1(x)\sin^2\frac{\theta}{2}\right]$$

Note:

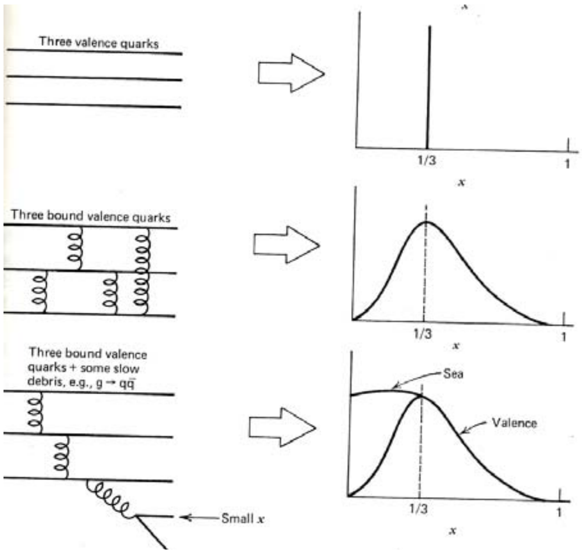
$$\frac{d\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4x}\left((1-y)F_2(x) + xy^2F_1(x)\right)$$

HERA

3.6

Interpretation of parton distribution function

Halzen - Martin



N.Herrmann, Uni Heidelberg

09/11/2011

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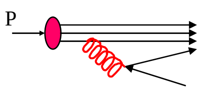
Quark composition of the proton



valence quarks

sea quarks

Quark composition of proton:

$$u_v + u_v + d_v + (u_s + \bar{u}_s) + (d_s + \bar{d}_s) + (s_s + \bar{s}_s)$$

Heavy quarks in ‘sea’ strongly suppressed.

Flavour composition of proton:

$$\frac{F_2^{ep}(x)}{x} = \sum_i e_i^2 q_i(x)$$
$$= \frac{4}{9}\left(u^p(x) + \bar{u}^p(x)\right) + \frac{1}{9}\left(d^p(x) + \bar{d}^p(x)\right) + \frac{1}{9}\left(s^p(x) + \bar{s}^p(x)\right)$$

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Quark composition of the nucleon

Flavour composition of neutron:

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}\left(u^n(x)+\bar{u}^n(x)\right)+\frac{1}{9}\left(d^n(x)+\bar{d}^n(x)\right)+\frac{1}{9}\left(s^n(x)+\bar{s}^n(x)\right)$$

Isospin symmetry:

$$u^n(x) = d^p(x) = d(x)$$
$$d^n(x) = u^p(x) = u(x)$$
$$s^n(x) = s^p(x) = s(x)$$
$$\bar{q}^n(x) = \bar{q}^p(x) = \bar{q}(x)$$

Neutron:

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}\left(d(x)+\bar{d}(x)\right)+\frac{1}{9}\left(u(x)+\bar{u}(x)\right)+\frac{1}{9}\left(s(x)+\bar{s}(x)\right)$$

Nucleon:

$$\begin{aligned} F_2^{eN} &= \tfrac{1}{2}(F_2^{ep}+F_2^{en}) \\ &= \tfrac{1}{2}x(\tfrac{1}{9}(d+\bar{d})+\tfrac{4}{9}(u+\bar{u})+\tfrac{1}{9}(s+\bar{s}) \\ &\quad +\tfrac{1}{9}(u+\bar{u})+\tfrac{4}{9}(d+\bar{d})+\tfrac{1}{9}(s+\bar{s})) \\ &= \tfrac{5}{18}x\cdot[u+\bar{u}+d+\bar{d}]+\tfrac{1}{9}x\cdot[s(x)+\bar{s}(x)] \\ &\approx \tfrac{5}{18}x\left(u+\bar{u}+d+\bar{d}\right) \end{aligned}$$

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Sum rules

$$\int_0^1 dx\,\left(u(x)-\bar{u}(x)\right) = 2$$

$$\int_0^1 dx\,\left(d(x)-\bar{d}(x)\right) = 1$$

$$\int_0^1 dx\,\left(q_s(x)-\bar{q}_s(x)\right) = 0$$

Momentum conservation:

$$\int_0^1 dx\;x\left(u(x)+\bar{u}(x)+d(x)+\bar{d}(x)\right) = \tfrac{18}{5}\int_0^1 dx\;F_2^{eN}(x) \overset{?}{\approx} 0.5$$

Experimental observation:
electrically charged partons carry 50% of proton momentum.

Remaining momentum is carried by gluons.

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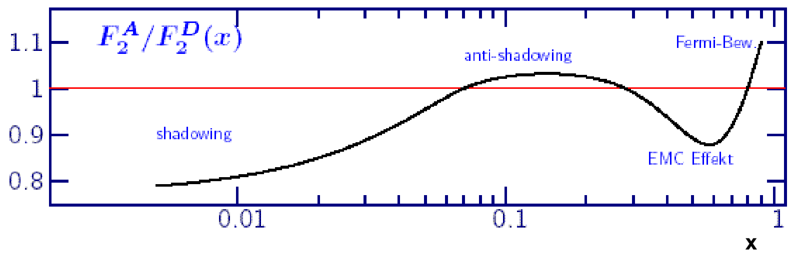
EMC – effect



$$F_2^A(x) \neq A \cdot F_2^N(x) \Rightarrow \text{EMC-effect}$$

A = 40 – 56 (Ca, Fe)

Kinematic regions:
- – shadowing: $x < 0.1$
- – enhancement: $0.1 < x < 0.2$
- – EMC-effect: $0.2 < x < 0.7$
- – Fermi-motion: $x > 0.7$