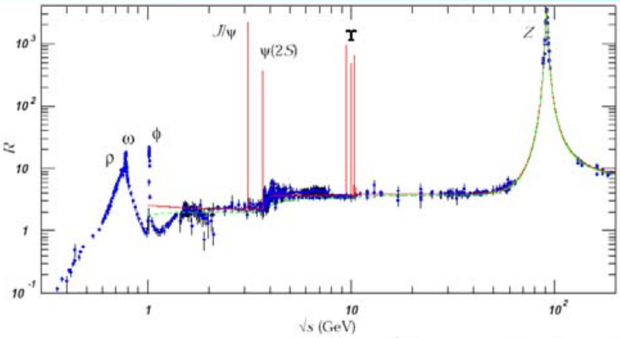
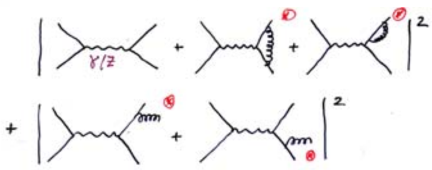
4.5

Electron – positron – hadron/muon ratio



Next order: $\sigma(e^+ + e^- \rightarrow hadrons) =$



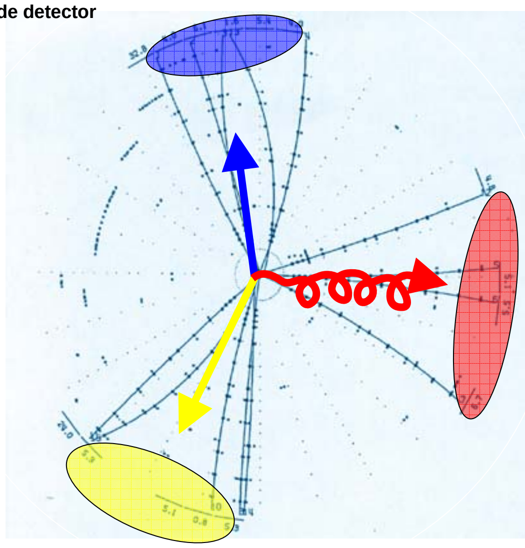
$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411\frac{\alpha_s^2(s)}{\pi^2} + \ldots\right]$$

$$R_{had} = 3\sum_f z_f^2\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411\frac{\alpha_s^2(s)}{\pi^2} + \ldots\right]$$

4.5

Gluon jets

Jade detector



Discovery

Petra (DESY) 1979

√s=31 GeV

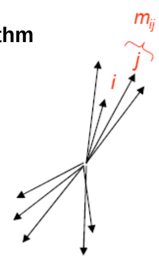
10% of all events are 3-jet events

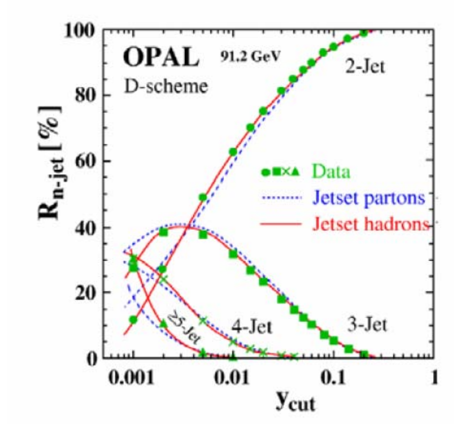
α_s can be determined from 3-jet/2-jet ratio

4.5

Jet definition

Example: Jade algorithm





Jet definition depends on algorithm!

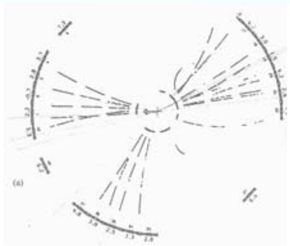
Hadronic particles i and j are grouped to a pseudoparticle k as long as the invariant mass m_{ij} is smaller than the jet resolution parameter y_{cut}:

$$\frac{m_{ij}^2}{E_{vis}} < y_{cut}$$

Remaining pseudo particles are jets.

4.5

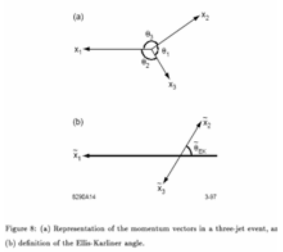
Spin of the gluon

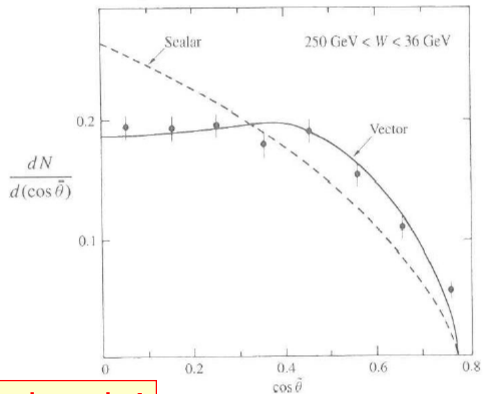


Ellis – Karliner – angle

1) order jet energies: $E_1 > E_2 > E_3$

2) measure direction of jet1
in rest frame of jet2 and jet3



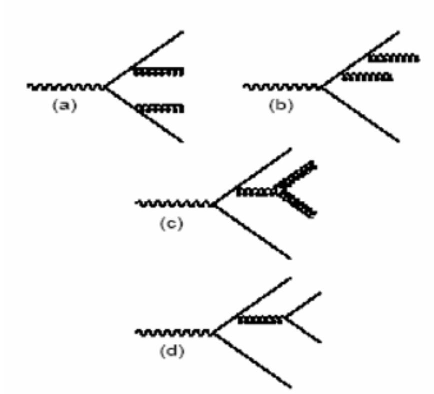


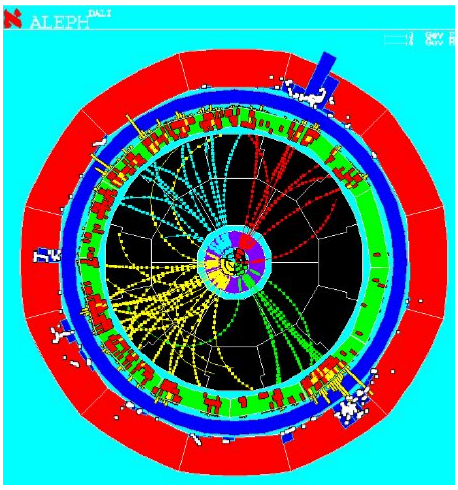
⇒ Gluon has spin 1

4.5

Triple gluon vertex

4 – jet events



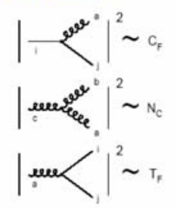


4.5

Angular distribution of 4-jet events

Group	N_C	C_F	T_F
U(1)	0	1	1
$U(1)_3$	0	1	3
SU(N)	N	$(N^2-1)/2N$	1/2
SU(3)	3	4/3	1/2

Color factors:

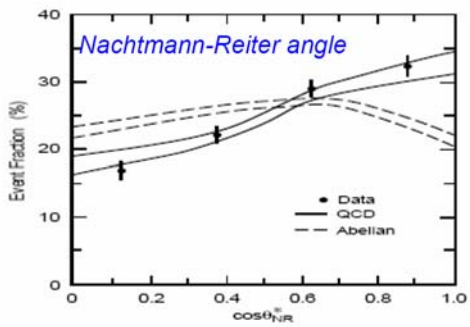


Sort jets according to energy:
$E_1 > E_2 > E_3 > E_4$

Nachtmann-Reiter angle

$$\cos\theta_{NR} \propto (\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)$$

CERN-EP/2002-23

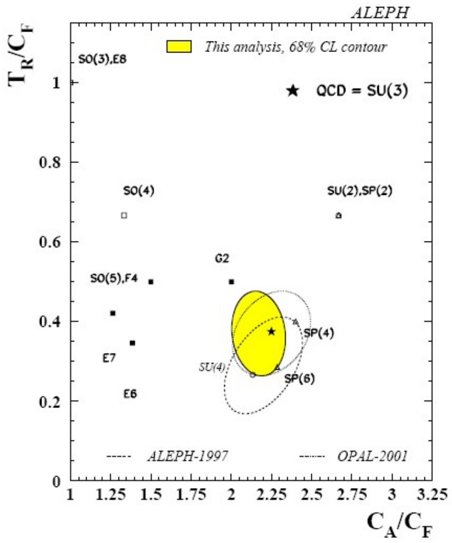


4.5

Color factors in SU(3)

Result of 4 – jet analysis

CERN-EP/2002-23



- measurement:

$$\frac{C_A}{C_F} = 2,17 \pm 0,06\,(stat) \pm 0,09\,(syst)$$

$$\frac{T_F}{C_F} = 0,37 \pm 0,02\,(stat) \pm 0,07\,(syst)$$

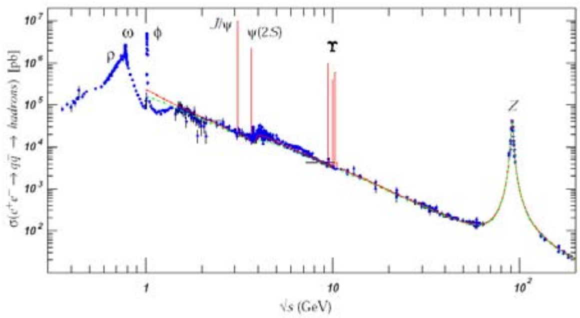
- theory:

$$\frac{C_A}{C_F} = 2,25\,(\mathrm{SU}(3)) \quad = 0 \quad \text{(abelsch)}$$

$$\frac{T_F}{C_F} = 0,375\,(\mathrm{SU}(3)) \quad = 3\,(U(1)_3)$$

4.6

QCD – bound states



Non – relativistic resonance form:

$$\sigma_{BW}\left(\sqrt{s}\right) = \frac{\pi}{|\vec{p}|^2} \frac{(2J+1)}{(2S_1+1)(2S_2+1)} \frac{\Gamma_i \Gamma_f}{\left(\sqrt{s}-m\right)^2 + \Gamma^2/4}$$

Partial width $\Gamma_i = B_i \Gamma$ **(B_i - branching fraction)**

Total width $\Gamma = \sum_i \Gamma_i$

4.6

Breit-Wigner resonance form

Stable particle in rest frame: $\Psi(t) = \Psi_0 \cdot e^{-iMt} \Rightarrow |\Psi(t)|^2 = |\Psi_0|^2 = const$

Particle with decay width Γ: $\Psi(t) = \Psi_0 \cdot e^{-iMt - \frac{\Gamma}{2}t} \Rightarrow |\Psi(t)|^2 = |\Psi_0|^2 \cdot e^{-\Gamma t}$

Energy dependence by Fourier transform:

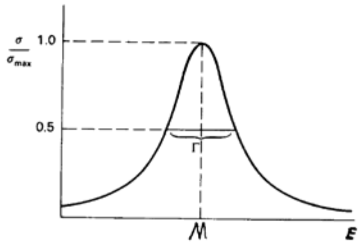
$$\tilde{\Psi}(E) = F(\Psi(t)) \sim \int_0^\infty \Psi(t) e^{iEt} dt \sim \int_0^\infty \Psi_0 e^{(iE - iM - \frac{\Gamma}{2})t} dt$$

$$\Downarrow$$

$$\tilde{\Psi}(E) \sim \frac{1}{(E-M) + i\frac{\Gamma}{2}}$$

$$|\tilde{\Psi}(E)|^2 \sim \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

Breit-Wigner form



$$\sigma(E) = \sigma_0 \frac{\frac{\Gamma^2}{4}}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

$\downarrow$ relativistically invariant form

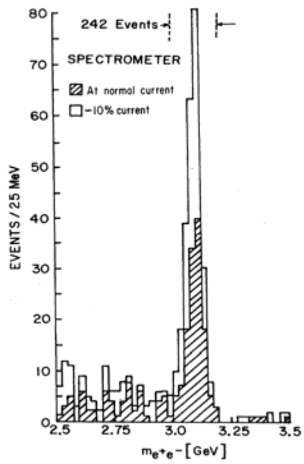
$$\sigma \sim \frac{M^2\Gamma^2}{(s-M^2)^2 + M^2\Gamma^2}$$

4.6

Discovery of J/Ψ (1974)

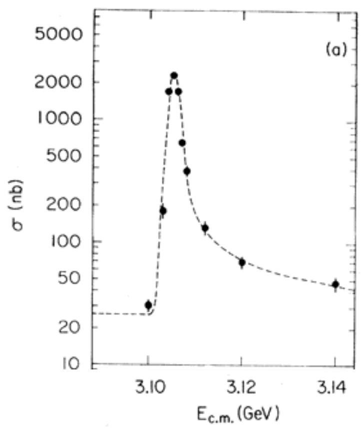
BNL (AGS): S.Ting et al.

$$p(28GeV) + Be \rightarrow e^- + e^+ + X$$



Invariant mass of detected electrons

SLAC: B. Richter et al.
e^+e^- - annihilation at SPEAR collider
(2.5 < √s < 7.5 GeV)
MARK I Detektor

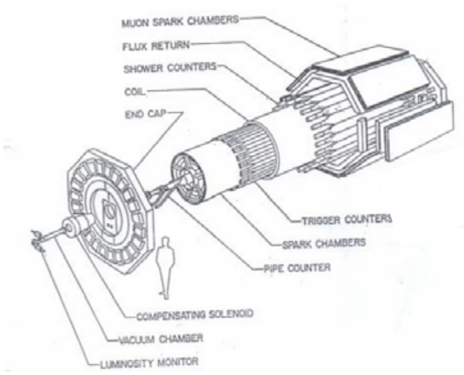


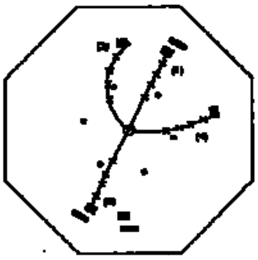
Cross section of (multi) hadronic events

4.6

Naming of J/Ψ

MARK-I Detector





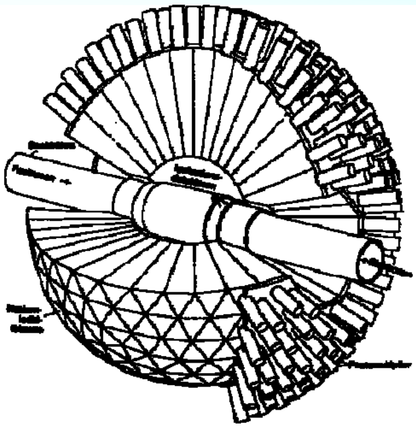
"Eventdisplay" of Mark I detector:
cut perpendicular to beam direction
magnetic field parallel to beam direction

$$e^+ + e^- \to J/\Psi(3700) \to J/\Psi(3100) + \pi^+(3) + \pi^-(4)$$
$$\mapsto \; e^+(1) + e^-(2)$$

cross section in e^+e^-

4.6

Quarkonium spectroscopy: Crystal ball detector

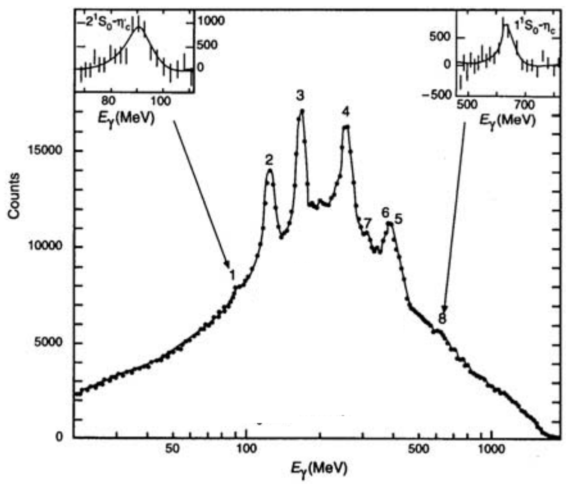


Photon – detection in full solid angle with NaI (Th) scintillators

4.6

Photon spectrum of Ψ (3700) – decays

Povh:13.3



Plausible assumption:
Dominance of electric dipole – transitions

$\Delta L=1,\ \Delta S=0$

Level scheme like positronium

$n\ ^{2S+1}L_j$

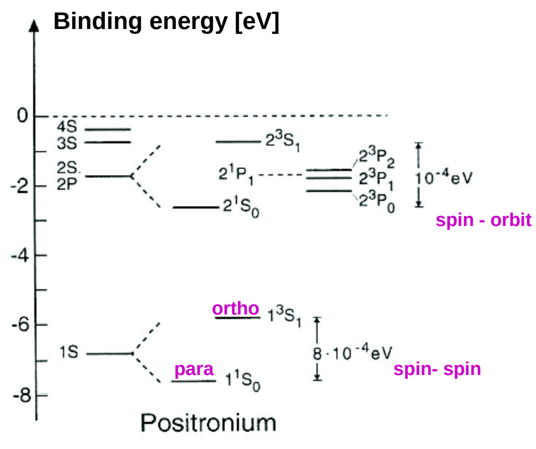
Quantum numbers

Parity $P = (-1)^{L+1}$

C- Parity $C = (-1)^{L+S}$

4.6

Positronium



Properties of Coulomb – potential:

Degeneracy of energy levels with same principal quantum numbers (2s, 2p)
2s - N=1,l=0
2p - N=0,l=1

Fine structure due to spin – orbit – interaction (ls – coupling)
$\Delta E \sim \alpha^2$

Hyperfine splitting due to spin – spin – interaction
$\Delta E \sim \alpha^2 \cdot \mu_{p/e} / \mu_e$.

Spin – orbit and spin – spin interaction are of similar strength.
Spectroscopy notation:
$n\ ^{2S+1}L_J$ (n=N+l+1)

Decay of positronium:

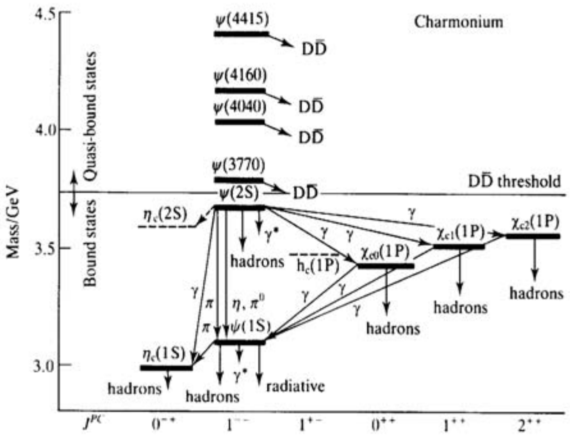
$$\Gamma \propto |M_{fi}|^2 \cdot |\Psi(0)|^2$$

$$\uparrow \text{overlap probability} \propto \frac{1}{r_B^3} = m^3\alpha^3$$

Para- positronium	$1\ ^1S_0 \rightarrow 2\gamma$,	$\Gamma = ½$	$m_e\ \alpha^5$,	τ=1.252 · 10^{-10} s
Ortho-positronium	$1\ ^3S_1 \rightarrow 3\gamma$,	$\Gamma = 2(\pi^2\text{-}9)/9\pi$ ·	$m_e\ \alpha^6$,	τ=1.374 · 10^{-7} s

4.6

Charmonium



$c\bar{c}$ – states are called charmonium.

in e^+e^- - annihilation only $J^{PC}=1^{--}$ - states are populated.

∃ 2 bound states below $D\bar{D}$ - threshold

Nomenclature:
$n\ {}^{2S+1}L_J$
n=N+1 (different to positronium)

J/Ψ (3100) = 1 $^3S_1$

Observations and conclusions:
2 ^3S and 1 ^3P – states not degenerate ⇒ potential is not a pure 1/r – potential
V(r) = - 4/3 α_s/r + kr
Strong splitting of S – states ⇒ colormagnetic interaction (spin-spin interaction)

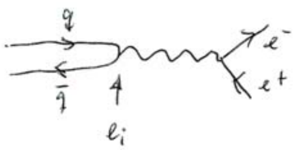
4.6

Vector – meson – decays

Leptonic width of vector mesons,

Van Royen – Weisskopf formula (1967):

$$\Gamma(V \to l^+l^-) = \frac{16\pi\alpha^2 Q_V^2}{M_V^2}|\psi(0)|^2\left(1-\frac{2m_l^2}{M_V^2}\right)\left(1-\frac{4m_l^2}{M_V^2}\right)^{\frac{1}{2}}$$



$$Q_V^2 = \left|\sum_i e_i\right|^2$$

Sum over all contributing quark flavors, use flavor wave functions.

Observation:

width governed by quark charge Q_V^2

phase space for all vectormesons similar

4.6

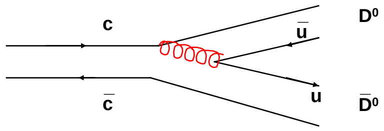
Decay of 1 $^3S_1$ – state

OZI – rule:

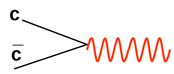
Suppression of diagrams with broken quark lines is called *OZI – rule* .
OZI – rule parametrizes small decay widths of quarkonia states with

$$m_{q\bar{q}} < 2m_{qx} (= \text{lightest meson with flavour q})$$

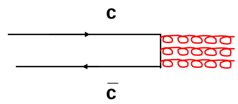
Ex.: $\Psi(3770) \rightarrow D_0 + \bar{D}_0$,
Γ_{tot} = 24 MeV



Γ(J/Ψ) = 87 keV
decay channels:
1. virtual photon



2. 3 – gluon decay (C – parity and color singlet)



Sensitivity to α_s:

$$\frac{\Gamma\left(1^3S_1 \rightarrow 3g \rightarrow \text{Hadronen}\right)}{\Gamma\left(1^3S_1 \rightarrow \gamma^* \rightarrow 2\text{Leptonen}\right)} \propto \frac{\alpha_s^3}{\alpha^2}$$

N.Herrmann, Uni Heidelberg 22/11/2011