

4.8 Spontaneous chiral symmetry breaking

M [GeV/c²]

$J^P=0^\pm$ 1^\pm $1/2^\pm$

Exact chiral symmetry:

all hadrons should exist in 2 degenerate parity states!

Parity operator $P = \gamma^0$:

$$P |q_R\rangle = |q_L\rangle$$

$$P |q_L\rangle = |q_R\rangle$$

$$P_R = \frac{1}{2}(1 + \gamma^5)$$

$$P_L = \frac{1}{2}(1 - \gamma^5)$$

Construct: $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|q_R\rangle \pm |q_L\rangle)$

$$P |\psi_+\rangle = + |\psi_+\rangle$$

$$P |\psi_-\rangle = - |\psi_-\rangle$$

These 2 states should have the same energy / mass.
 This feature is not observed in nature,
 Mass difference between chiral partners much larger than current quark mass difference

⇒ chiral symmetry is spontaneously broken.

Chiral condensate fills QCD vacuum:

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$

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4.8 Symmetry breaking

- **spontaneous breaking of electro-weak symmetry (Higgs mechanism)**
 ⇒ **current mass of quark**
 - for **u & d quarks** $m_u^0 \sim m_d^0 \sim 5 \text{ MeV}$
 - s quark** $m_s^0 \sim 175 \text{ MeV}$

explicitly breaking of chiral symmetry

- **spontaneous breaking of chiral symmetry**
 ⇒ **constituent mass of quarks**

q couples to $\bar{q}q$ vacuum

coupling G

for **u & d quarks**

s quark

$$m_q = m_q^0 - G \langle \bar{q}q \rangle$$

$$m_u \sim m_d \sim 300 \text{ MeV} \quad (\sim 1/3 m_{\text{proton}})$$

$$m_s^0 \sim 500 \text{ MeV}$$

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4.8 Consequences of Spontaneous Chiral Symmetry Breaking

- 1) All hadrons have well defined parity, chiral J^P doublets not observed.

1^\pm

- 2) Chiral symmetry spontaneously broken, vacuum is filled with qq - condensate.
- 3) Goldstone theorem: Any spontaneously broken continuous symmetry generates a massless boson (\rightarrow Goldstone bosons).
- 4) Characteristic mass scale of hadrons
1 GeV mass gap to quark condensate

except pseudoscalar mesons that are the Goldstone bosons:
 $\pi, \eta,$ and K

mass (GeV)

"Gap" $\Delta \sim 1$ GeV

quark condensate $\langle qq \rangle$

pseudoscalar mesons ($J^P = 0^-$)

$\pi, \eta, K, \rho, \omega, \eta', \phi, K^*$

$B=0, S=0, 1$ and $B=1, S=0, 1$ hadrons: $N, \Lambda, \Sigma, \Delta$

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4.8 Chiral symmetry restoration of QCD

Chiral Condensate

W.Weise, Prog. Theor. Phys. Suppl. 149 (2003) 1
initially: S.Klimt et al., PLB 249, 386 (1990)

Chiral symmetry should be restored at sufficiently high temperatures and baryon densities.

Reduction of vacuum value should be visible already at moderate densities (\rightarrow partial chiral symmetry restoration)

Symmetry breaking pattern of Chiral Symmetry of QCD

Gell-Mann-Oaks-Renner Relation:

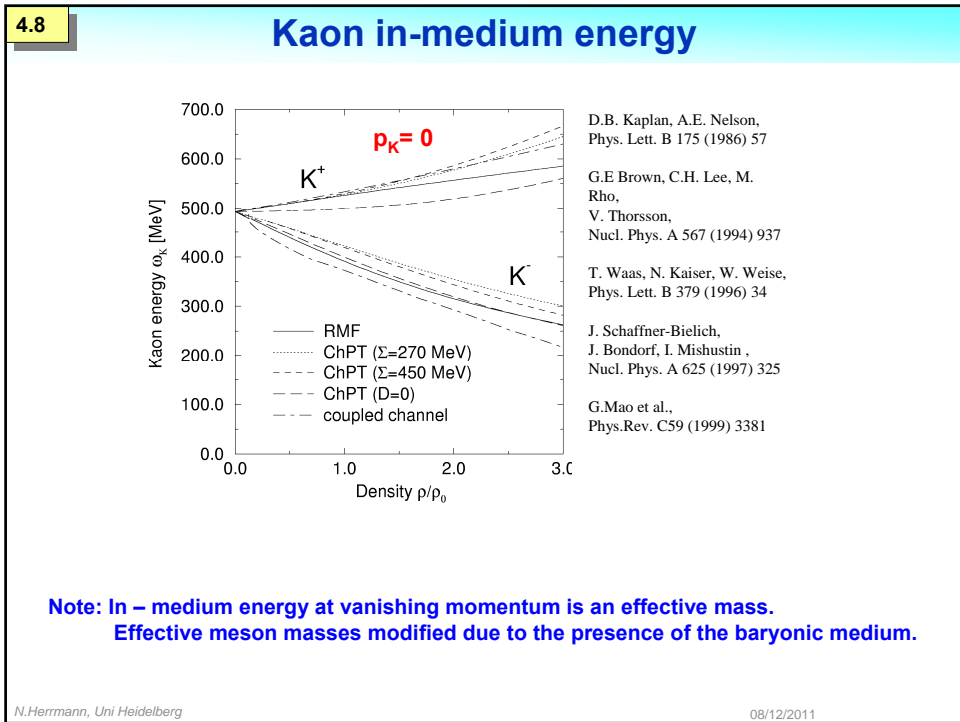
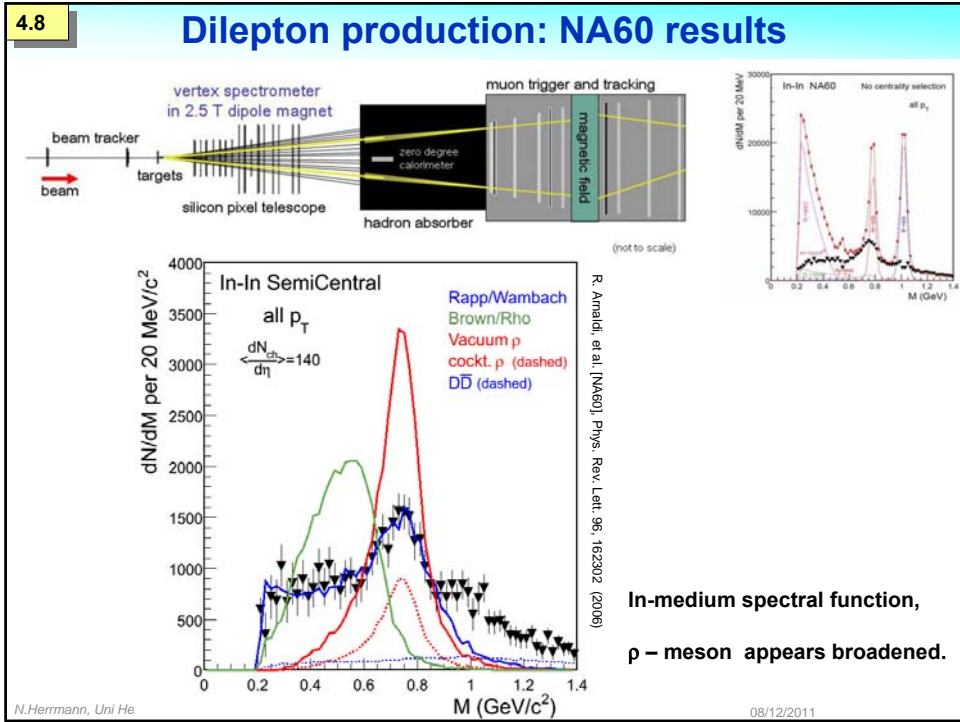
$$m_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + O(m_q^2)$$


$$m_K^2 f_K^2 = -\frac{1}{2}(m_u + m_s) \langle \bar{u}u + \bar{s}s \rangle + O(m_s^2)$$

spontaneous symmetry breaking

explicit symmetry breaking

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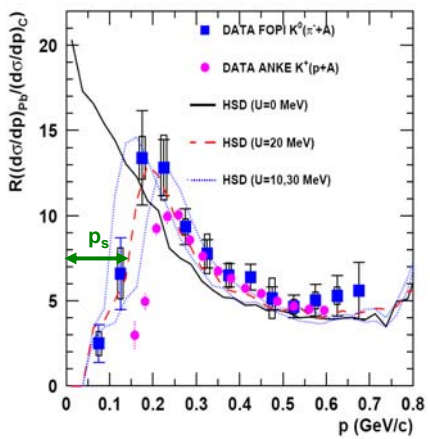


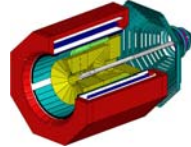
4.8 

K⁰ production in π⁻ + A reactions at 1.15 GeV/c

M.L. Benabderramane et al., PRL 102, 182501 (2009), arXiv:0807.3361

Ratio of momentum distributions:





Anke data @ COSY
M. Büscher et al., EPJ A22, 301 (2004)
p + p → K⁺ + X at 2.5 GeV

Model interpretation with RBUU
U(K⁺) = + 20 MeV

Model independent analysis:

$$U_K = \frac{p_s^2}{2m_K} = \frac{(140\text{MeV})^2}{2 \cdot 498\text{MeV}} = 20\text{MeV}$$

Potential depth: U(K⁰) = + 20 (+/- 5) MeV consistent with heavy-ion data on K⁺, Accuracy (only) statistics limited,

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5.

Weak interactions

Characteristica of weak processes:

- long lifetimes, e.g. $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ with $\tau = 2.8 \cdot 10^{-8} s$
- neutrinos
- parity violation
- violation of flavor conservation: $\Lambda \rightarrow \pi^- + p$ ($\Delta S=1$) with $\tau = 2.6 \cdot 10^{-10} s$

Observation of weak interaction only possible, if strong and electromagnetic interactions are forbidden. (or via interference of effects: γ/Z – interference, e.g. atomic parity violation)

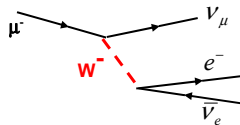
Summary of conservation laws

	strong	e.m.	weak
Flavor (strangeness, charm, beauty, top)	yes	yes	no
(strong) isospin I ₃	yes	yes	no
Lepton number: L _e , L _μ , L _τ	-	yes	(yes)
Baryon number	yes	yes	yes

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5.1 Phenomenology

Leptonic decays $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$
with $\tau = 2.6 \cdot 10^{-6} s$

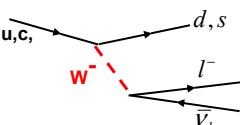


Hadronic decays:

dominant modes

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

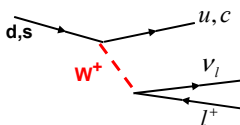
$$u \rightarrow d + l^+ + \nu_l \quad c \rightarrow s + l^+ + \nu_l$$

$$d \rightarrow u + l^- + \bar{\nu}_l \quad s \rightarrow c + l^- + \bar{\nu}_l$$


suppressed modes

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

$$u \rightarrow s + l^+ + \nu_l \quad c \rightarrow d + l^+ + \nu_l$$

$$d \rightarrow c + l^- + \bar{\nu}_l \quad s \rightarrow u + l^- + \bar{\nu}_l$$


If Q^2 large enough, also:

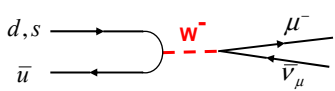
$d \rightarrow u + l^- + \bar{\nu}_l \quad \sim \cos^2 \theta_C \sim 0.95$

$u \rightarrow d + l^+ + \nu_l \quad \sim \sin^2 \theta_C \sim 0.05$

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5.1 Phase space estimate for pseudoscalar decays

Compare π^- to K^- decays



$$\Gamma_{\mu \bar{\nu}_\mu}(\pi^-) = \frac{1}{\tau} = 3.8 \cdot 10^8 s^{-1}$$

$$\Gamma_{\mu \bar{\nu}_\mu}(K^-) = BR_l \cdot \Gamma = BR_l \cdot \frac{1}{\tau} = 5.3 \cdot 10^7 s^{-1}$$

Decay width of kaon smaller than pion despite larger mass.

Phase space (from chapter 1.3):

$$\Gamma_{\beta} = \frac{1}{32\pi^2 m_i} \int |M_{\beta}|^2 \left| \frac{E_1 E_2}{|\vec{p}_1| (E_1 + E_2) E_1 E_2} \frac{|\vec{p}_1|^2}{|\vec{p}_1|} \right|_{p_1=p} d\Omega$$

$$= \frac{|\vec{p}^*|}{32\pi^2 m_i^2} \int |M_{\beta}|^2 d\Omega$$

Phase space ratio:

$$\frac{|\vec{p}^*|}{m_i}(\pi^-) = \frac{30}{140} = 0.21$$

$$\frac{|\vec{p}^*|}{m_i}(K^-) = \frac{236}{494} = 0.48$$

$$R_{\text{phase space}}\left(\frac{\pi^-}{K^-}\right) = 0.44$$

$$R_{\text{exp}}\left(\frac{\Gamma(\pi^-)}{\Gamma(K^-)}\right) = 7.2 > R_{\text{phase space}}\left(\frac{\pi^-}{K^-}\right) = 0.44$$

\Rightarrow **K - decay suppressed**

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