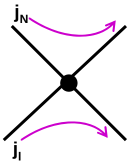
5.3

Fermi – theory of weak interactions



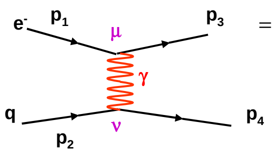
Ansatz for neutron decay: $n \longrightarrow p e^- \overline{\nu}_e$

Fermi (1934): construct like e.m. interaction

$$M = G_F\, j_N \cdot j_e = G_F \left(\overline{u}_p \gamma^\mu u_n\right)\left(\overline{u}_e \gamma_\mu \mathrm{v}_\nu\right)$$

However, parity is conserved in e.m. interactions.

$$M_{fi} = \left[\overline{u}_e(p_3) e_e \gamma^\mu u_e(p_1)\right] \frac{-g_{\mu\nu}}{q^2} \left[\overline{u}_q(p_4) e_q \gamma^\nu u_q(p_2)\right]$$



$$= -e_e e_q \frac{j_e \cdot j_q}{q^2}$$

$$\hat{P} = \gamma^0$$
$$\hat{P}(u) = \gamma^0 u$$
$$\hat{P}(\overline{u}) = \overline{u}\gamma^0$$

$$\hat{P}(j_e) = \overline{u}_e(p_3)\gamma^0\gamma^\mu\gamma^0 u_e(p_1) \rightarrow \begin{cases} \hat{P}(j_e^0) = \overline{u}_e(p_3)\gamma^0 \underbrace{\gamma^0\gamma^0}_{1} u_e(p_1) = j_e^0 \\ \hat{P}(j_e^k) = \overline{u}_e(p_3)\gamma^0 \underbrace{\gamma^k\gamma^0}_{-\gamma^0\gamma^k} u_e(p_1) = -j_e^k \end{cases}$$

$$\hat{P}(j_q) = \ldots\text{(correspondingly)}$$
$$\hat{P}(j_e \cdot j_q) = \hat{P}\left(j_e^0 j_q^0 - j_e^k j_q^k\right) = j_e^0 j_q^0 - \left(-j_e^k\right)\left(-j_q^k\right) = j_e \cdot j_q$$

Ansatz cannot explain parity violation.

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Bilinear covariants

Lorentz invariant currents

Weak interactions must have a structure different from e.m. vector to account for parity violation.

Limited number of Lorentz invariant currents available (called "bilinear covariants")

SCALAR	$\overline{\Psi}\phi$
PSEUDOSCALAR	$\overline{\Psi}\gamma^5\phi$
VECTOR	$\overline{\Psi}\gamma^\mu\phi$
AXIAL VECTOR	$\overline{\Psi}\gamma^\mu\gamma^5\phi$
TENSOR	$\overline{\Psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$

Scalar and pseudoscalar interaction	-> exchange of	spin=0 bosons
Vector and axial vector	->	spin=1
Tensor	->	spin=2

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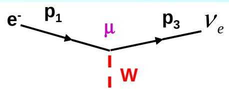
V-A – structure of weak interaction

From experiment: **parity violation**
neutrino helicity
muon decay



The form of weak interaction is V – A:

$$j^{\mu} = \overline{u}_{\nu_e}(p_3)\left(\gamma^{\mu} - \gamma^{\mu}\gamma^{5}\right)u_e(p_1)$$

V A

Pure axial vectors conserve parity

$$j_A = \overline{\psi}\gamma^{\mu}\gamma^{5}\phi$$

$$\hat{P}(j_A) = \hat{P}\left(\overline{\psi}\gamma^{\mu}\gamma^{5}\phi\right) = \overline{\psi}\gamma^{0}\gamma^{\mu}\gamma^{5}\gamma^{0}\phi \rightarrow \begin{cases} \hat{P}\left(j_A^0\right) = \overline{\psi}\gamma^{0}\gamma^{0}\gamma^{5}\gamma^{0}\phi = -\overline{\psi}\gamma^{0}\gamma^{0}\gamma^{0}\gamma^{5}\phi = -j_A^0 \\ \hat{P}\left(j_A^k\right) = \ldots = j_A^k \end{cases}$$

$$\hat{P}(j_{A1}\cdot j_{A2}) = j_{A1}\cdot j_{A2}$$

Combination of vector and axial vector

$$\hat{P}(j_{V1}\cdot j_{A2}) = \left(j_1^0\right)\left(-j_2^0\right) - \sum_{k=1,3}\left(-j_1^k\right)\left(j_2^k\right) = -j_{V1}\cdot j_{A2}$$

parity violating

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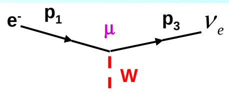
Helicity structure of weak interaction

Charged current weak vertex can be written as

$$\frac{-ig_w}{\sqrt{2}}\gamma^{\mu}\underbrace{\frac{1}{2}\left(1-\gamma^{5}\right)}_{P_L}$$



Implications:

$$\overline{\psi}\gamma^{\mu}\underbrace{\frac{1}{2}\left(1-\gamma^{5}\right)}_{P_L}\phi = \overline{\psi}\gamma^{\mu}\phi_L$$

$$= \left(\overline{\psi}_R + \overline{\psi}_L\right)\gamma^{\mu}\phi_L$$

$$\downarrow \overline{\psi}_R\gamma^{\mu}\phi_L = \tfrac{1}{2}\psi^{+}\left(1+\gamma^{5}\right)\gamma^{0}\gamma^{\mu}\,\tfrac{1}{2}\left(1-\gamma^{5}\right)\phi = \tfrac{1}{4}\psi^{+}\gamma^{0}\left(1-\gamma^{5}\right)\gamma^{\mu}\left(1-\gamma^{5}\right)\phi$$

$$\downarrow \qquad = \tfrac{1}{4}\overline{\psi}\gamma^{\mu}\left(1+\gamma^{5}\right)\left(1-\gamma^{5}\right)\phi = 0$$

$$= \overline{\psi}_L\gamma^{\mu}\phi_L$$

⇨ **Only the left – handed chiral components of particle spinors and right – handed chiral components of anti – particle spinors participate in charged current weak interactions.**

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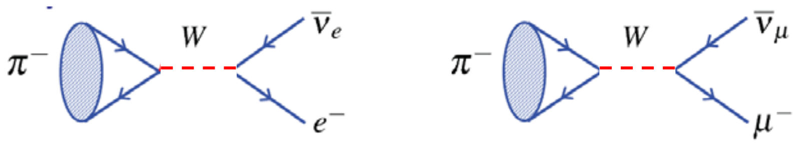
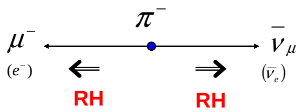
Helicity in pion – decay



Momentum and angular momentum conservation:



Phase space favors electron channel.

But:
Since anti-neutrino is (almost) massless, CC weak interaction can only occur in RH state.

Myon (electron) has to be in RH state as well.
Weak interaction will couple to LH chiral component of RH helicity state.

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Charged lepton spinor

Weak interaction will couple to LH chiral component of RH helicity state:

Right handed lepton spinor:

$$P_L u_\uparrow = \tfrac{1}{2}(1-\gamma^5)u_\uparrow = \tfrac{1}{2}N\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\frac{\vartheta}{2} \\ e^{i\varphi}\sin\frac{\vartheta}{2} \\ \frac{|\vec{p}|}{E+m}\cos\frac{\vartheta}{2} \\ \frac{|\vec{p}|}{E+m}e^{i\varphi}\sin\frac{\vartheta}{2} \end{pmatrix} = \tfrac{1}{2}N\left(1-\frac{|\vec{p}|}{E+m}\right)\underbrace{\begin{pmatrix} \cos\frac{\vartheta}{2} \\ e^{i\varphi}\sin\frac{\vartheta}{2} \\ -\cos\frac{\vartheta}{2} \\ -e^{i\varphi}\sin\frac{\vartheta}{2} \end{pmatrix}}_{u_L}$$

Right handed helicity spinor has left handed chiral component.

Matrix element: $M_{fi} \propto \tfrac{1}{2}\left(1-\frac{|\vec{p}|}{E+m}\right)$

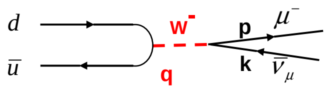
Polarisation from: $u_\uparrow = P_R u_\uparrow + P_L u_\uparrow = \tfrac{1}{2}N\left(1+\frac{|\vec{p}|}{E+m}\right)u_R + \tfrac{1}{2}N\left(1-\frac{|\vec{p}|}{E+m}\right)u_L$

RH helicity RH chiral LH chiral

$$Pol = \frac{\langle P_R\rangle - \langle P_L\rangle}{\langle P_R\rangle + \langle P_L\rangle} = -\beta$$

5.3

Branching ratio in pion – decay



L.I. Matrix element:

$$M = \frac{G}{\sqrt{2}} \underbrace{(\ldots)_\mu}_{\text{bound state}} \bar{u}(p)\gamma^\mu(1-\gamma^5)v(k)$$

$$\Downarrow \quad \underbrace{(\ldots)^\mu}_{\text{bound state}} = q^\mu f(q^2) = q^\mu f_\pi$$

$$M = \frac{G}{\sqrt{2}} f_\pi m_\mu \bar{u}(p)(1-\gamma^5)v(k)$$

$$\overline{|M|^2} = 4G^2 f_\pi^2 m_\mu^2 (p\cdot k)$$

$$\Downarrow \ldots\text{integrating over LIPS}\ldots$$

Decay rate:

$$\Gamma = \frac{1}{\tau} = \frac{G^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$\Downarrow$$

$$R = \frac{\Gamma(\pi^- \to e^- \nu_e)}{\Gamma(\pi^- \to \mu^- \nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.283\cdot 10^{-4}$$

$$R_{\exp} = (1.230 \pm 0.004)\cdot 10^{-4}$$

excellent agreement

5.3

Amplitude of charged current interaction

Propagator for exchange of massive particle: $\frac{1}{q^2 - m^2}$

W – boson propagator:
Spin 1 $W^{+/-}$
(Nominator is modified due to W – polarisation states)

$$\frac{-i[g_{\mu\nu} - q_\mu q_\nu / m_W^2]}{q^2 - m_W^2}$$

W – boson propagator for $q^2 << m_W^2$:
Pointlike interaction

$$\frac{i g_{\mu\nu}}{m_W^2}$$

Matrix - elements
Fermi – theory (1934):
with G_F=1.166 · 10^{-5} GeV^{-2}

$$M_{fi} = G_F(\bar{u}_p \gamma^\mu u_n)(\bar{u}_e \gamma_\mu v_\nu)$$

Point interaction including parity violation(1957):
√2 to keep numerical value of G_F

$$M_{fi} = \frac{G_F}{\sqrt{2}}(\bar{u}_p \gamma^\mu (1-\gamma^5) u_n)(\bar{u}_e \gamma^\nu (1-\gamma^5) v_\nu)$$

W – boson exchange

$$M_{fi} = \left(\frac{g_W}{\sqrt{2}} \bar{u}_p \tfrac{1}{2}\gamma^\mu (1-\gamma^5) u_n\right) \frac{g_{\mu\nu} - q_\mu q_\nu / m_W^2}{q^2 - m_W^2} \left(\frac{g_W}{\sqrt{2}} \bar{u}_e \gamma^\nu \tfrac{1}{2}(1-\gamma^5) v_\nu\right)$$

$$\downarrow q^2 << m_W^2$$

$$M_{fi} = \frac{g_W^2}{8 m_W^2} g_{\mu\nu} (\bar{u}_p \gamma^\mu (1-\gamma^5) u_n)(\bar{u}_e \gamma^\nu (1-\gamma^5) v_\nu)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}$$

Note: G_F is still taken as a measure of the strength of weak int. as it can be accurately measured in μ – decays.

Cross section estimate

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Strength of weak interaction

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

Muon decay: $G_F = 1.16639(1) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$

W – boson mass: $m_W = 80.403 \pm 0.029\ \mathrm{GeV}$

$$\downarrow$$

Coupling constant: $\alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_F}{4\sqrt{2}\pi} = \frac{1}{29.5}$

The intrinsic strength of the weak interaction is greater than the e.m. interaction.

Mass of W – boson makes weak interactions appear weak.

=> for $q^2 >> m_W^2$ weak interactions are more likely than e.m. interactions.

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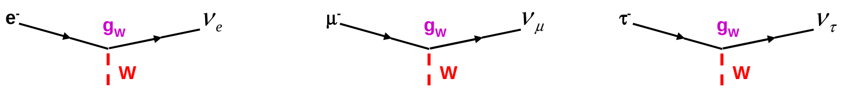
Lepton universality

From lepton decays (comparison of partial decay widths):

$$G_F^\tau = G_F^\mu = G_F^e$$



Charged current lepton universality

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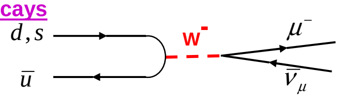
Weak interaction of quarks

π^- / K^- decays



$$\Gamma_{\mu^-\bar{\nu}_\mu}(\pi^-) = \frac{1}{\tau} = 3.8\cdot 10^8 s^{-1}$$

$$\Gamma_{\mu^-\bar{\nu}_\mu}(K^-) = BR_i \cdot \Gamma = BR_i \cdot \frac{1}{\tau} = 5.3\cdot 10^7 s^{-1}$$

$\Rightarrow$ **K - decay suppressed**

$$\Gamma_\pi = \frac{1}{\tau} = \frac{G^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 = \frac{G^2}{8\pi} \underbrace{f_\pi^2 m_\pi^2}_{\text{meson bound state}} \underbrace{\frac{m_\mu^2}{m_\pi}\left(m_\pi^2 - m_\mu^2\right)^2}_{\text{phase space}}$$

$$\frac{\Gamma_K}{\Gamma_\pi} = \frac{\frac{m_\mu^2}{m_K}\left(m_K^2 - m_\mu^2\right)^2}{\frac{m_\mu^2}{m_\pi}\left(m_\pi^2 - m_\mu^2\right)^2} = \frac{m_\pi\left(m_K^2 - m_\mu^2\right)^2}{m_K\left(m_\pi^2 - m_\mu^2\right)^2} = 17.7$$

Cabibbo (1963):

weak eigenstates are different from mass eigenstates,
weak interactions of quarks have same strength as for leptons
but a u – quark couples with the universal strength to a linear combination of s and d, the d' state.

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

weak eigenstates **strong (mass) eigenstates**

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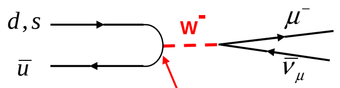
Cabbibo – angle



modified coupling:
g · cos θ_c for d – quark
g · sin θ_c for s – quark

$$\frac{\Gamma(K^+ \to \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} = \frac{\sin^2\theta_C}{\cos^2\theta_C}\left(\frac{m_K}{m_\pi}\right)\left(\frac{1 - \left(\frac{m_\mu}{m_K}\right)^2}{1 - \left(\frac{m_\mu}{m_\pi}\right)^2}\right)^2$$

Experimentally (PDG 2011):
sin θ_c = 0.225 +/- 0.001
cos θ_c= 0.974
tan² θ_c= 0.053
cot² θ_c= 18.8